Name ____ Test 1, Spring 2022

Part 1: Basic Knowledge (5 points each, 20 points total)
For each problem, give a precise definition.

1) What does it mean for an integer to be <u>even</u>?

2) Let A and B be sets in some fixed universe U. What is the <u>intersection</u> of A and B?

3) Let A be a set. What does the notation $x \in A$ mean?

4) What is a statement?

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

5) Find the truth table for $(P \wedge Q) \Rightarrow R$ where P, Q, and R are statements.

6) Find the negation of:

$$\forall_{x \in \mathbb{Z}} \exists_{y \in \mathbb{Z}} (xy + y = x^2)$$

7) Draw a Venn Diagram illustrating the set $(A \cap B) \cup C$

8) What $\big([4,7] \cup (5,9)\big) \cap \mathbb{Z}$?

Part 4: Proofs (10 points each, 60 points total)

9) Let P, Q, and R be statements. Prove that:

$$\Big((P \Rightarrow (Q \Rightarrow R)) \wedge (P \Rightarrow Q) \wedge P \Big) \Rightarrow R$$

10) Let n be an even integer. Prove that n^2 is even.

11) Let A and B be sets. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

12) Let n be an integer. If $6|n$, prove that $3|n$.

13) Prove that for every natural number $n, \frac{1}{n} \leq 1$.

(We are not including 0)

14) Prove that $\sqrt{5}$ is irrational … just kidding. We'll save that for later. Instead, prove that there exists a rational number.