Name ____ Test 1, Spring 2022

Part 1: Basic Knowledge (5 points each, 20 points total)

For each problem, give a precise definition.

1) What does it mean for an integer to be <u>even</u>?

n is even iff $n = 2k$ for some $k \in \mathbb{Z}$.

2) Let A and B be sets in some fixed universe U. What is the <u>intersection</u> of A and B?

$A \cap B \coloneqq \{x \in U | x \in A \wedge x \in B\}$

3) Let A be a set. What does the notation $x \in A$ mean?

x is an element of A, meaning that x is a member of the set A.

4) What is a statement?

A sentence, expression, or some conglomeration of symbols that is, in theory, either true or false (but not both).

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

5) Find the truth table for $(P \wedge Q) \Rightarrow R$ where P, Q, and R are statements.

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

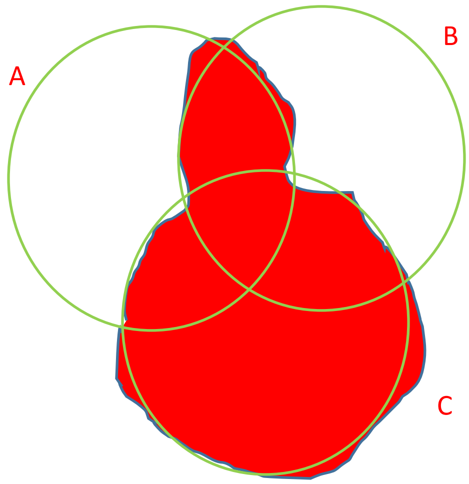
6) Find the negation of:

$$\forall_{x \in \mathbb{Z}} \exists_{y \in \mathbb{Z}} (xy + y = x^2)$$

$\exists_{x \in \mathbb{Z}} \forall_{y \in \mathbb{Z}} (xy + y \neq x^2)$

7) Draw a Venn Diagram illustrating the set $(A \cap B) \cup C$

Plz excuse the drawing…



8) What $\left([4,7] \cup (5,9)\right) \cap \mathbb{Z}$?

$[4,9) \cap \mathbb{Z} = \{4,5,6,7,8\}$

Part 4: Proofs (10 points each, 60 points total)

9) Let P, Q, and R be statements. Prove that:

$$\big((P \Rightarrow (Q \Rightarrow R)) \wedge (P \Rightarrow Q) \wedge P\big) \Rightarrow R$$

1 P	Premise
2 $P \Rightarrow Q$	Premise
3 $P \Rightarrow (Q \Rightarrow R)$	Premise
4 Q	Modus Ponens on lines 1 and 2.
5 $Q \Rightarrow R$	Modus Ponens on lines 1 and 3.
6 R	Modus Ponens on lines 4 and 5.

7 Therefore, $\big((P \Rightarrow (Q \Rightarrow R)) \wedge (P \Rightarrow Q) \wedge P\big) \Rightarrow R$

10) Let n be an even integer. Prove that n^2 is even.

1 Assume n is an even integers.	
2 $\therefore n = 2k$ for some $k \in \mathbb{Z}$	Definition of even
3 $\therefore n^2 = 4k^2 = 2(2k^2)$	Algebra
4 $\therefore n^2$ is even	Definition of even.

11) Let A and B be sets. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

1 Assume $A \subseteq B$	
2 Assume $X \in \mathcal{P}(A)$	
3 $\therefore X \subseteq A$	Definition of power set
4 $\therefore X \subseteq B$	Transitive property of subset (T65)
5 $\therefore X \in \mathcal{P}(B)$	Definition of power set
6 $\therefore \mathcal{P}(A) \subseteq \mathcal{P}(B)$	Definition of subset

12) Let n be an integer. If $6|n$, prove that $3|n$.

1 Assume $6\|n$	
2 $\therefore n = 6k$ for some $k \in \mathbb{Z}$	Definition of divides
3 $\therefore n = 3(2k)$	Algebra on 2
4 $\therefore 3\|n$	Definition of divides

13) Prove that for every natural number $n, \frac{1}{n} \leq 1$.

(We are not including 0)

1 Let n be a natural number	
2 $\therefore n \geq 1$	Definition of natural numbers
3 $\therefore \frac{n}{n} \geq \frac{1}{n}$	Divide (2) by n, note that $n > 0$ so the sign doesn't flip.
4 $\therefore 1 \geq \frac{1}{n}$	Algebra on (3)
5 $\therefore \frac{1}{n} \leq 1$	Rewrite (4)

14) Prove that $\sqrt{5}$ is irrational … just kidding. We'll save that for later. Instead, prove that there exists a rational number.

Note that $2, 3 \in \mathbb{Z}$, therefore by the definition of rational number $\frac{2}{3} \in \mathbb{Q}$.