Name ____ Test 2, Spring 2022

Part 1: Basic Knowledge (5 points each, 20 points total)
For each problem, give a precise definition.

1) Consider the function notation below. What is "$x \mapsto f(x)$" called?

$$\begin{aligned} f : A &\to B \\ x &\mapsto f(x) \end{aligned}$$

2) Let S be a set. What is the definition of a <u>relation</u> on S?

3) Let $f: A \to B$ be a function. What does it mean for f to be <u>onto</u>? Give the definition.

4) Let R be a relation on the set S. What does it mean for R to be <u>symmetric</u>? Give the definition.

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

5) Find the indexed union below.

$$\bigcup_{k=1}^{\infty}\left[\frac{1}{k}, 2+\frac{1}{k}\right]$$

6) Evaluate the function below at 5.

$$\begin{aligned} f: \mathbb{R} &\to \mathbb{R}^2 \\ x &\mapsto (3x+2, x^2) \end{aligned}$$

7) Illustrate the relation below as a digraph. (The one with circular nodes and arrows between them)

$$\{(1,3),(1,4),(2,3),(3,4),(4,4)\}$$

8) Which of the following relations are symmetric? Circle Y for Yes and N or No.

Y N a) R on $\mathbb{R}$ given by xRy iff $xy = 5$.

Y N b) R on the set of polynomials given by fRg iff $f(0) \leq g(0)$

Y N c) R on $\mathbb{Z}$ given by xRy iff $x - 2y = 0$

Y N d) R on $\mathbb{Z}$ given by xRy iff $5|x - y$

Y N e) R on the set of people given by pRq iff p and q have the same last name.

Part 3: Proofs (10 points each, 40 points total)

9) Consider the relation below. Prove that it is reflexive.

Let S be the set of all polynomials. Define R as the relation on S given by:

$$fRg \text{ iff } \deg(f) \leq \deg(g)$$

10) Prove that the function below is injective (also known as one-to-one):

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 14x - 3 \end{aligned}$$

11) Let I be an index set and A_k a set for each $k \in I$. Prove that:

$$\forall_{j \in I} \left(A_j \subseteq \bigcup_{i \in I} A_i \right)$$

12) Prove that the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x$ is invertible.

Part 4: Review (20 points total)

13) Determine whether these statements are true or false.
(5 points)

T F a) $\forall_{x\in\mathbb{R}}(x^2 \geq 4)$
T F b) $\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(x + y = 4)$
T F c) $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 0)$
T F d) $\exists_{x\in\mathbb{Z}}(6|x)$
T F e) $\forall_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}\forall_{z\in\mathbb{R}}(xyz = 1)$

14) Find $[3,5] \cap (4,6)$.
(5 points)

15) If x is an even integer and y is an odd integer, prove that $2x + y$ is odd.
(10 points)