

Name _____ Test 3, Spring 2022

Part 1: Basic Knowledge (5 points each, 10 points total)

1) What does it mean for $a \equiv_n b$? State the definition.

2) What does it mean addition to be well defined mod n ? State the definition.

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

3) Find $3 \cdot 6 - 4 \pmod{10}$.

4) Solve $3x \equiv 5 \pmod{7}$.

5) Solve $3x \equiv 6 \pmod{12}$.

6) What is $[5]_{20}$? No words please. Just math.

Part 3: Proofs (50 points total)

7) Prove that $[5]_{10} \cap [6]_{10} = \emptyset$

(It is not enough to write them down and point to it, though that would get you partial credit. *Prove* that they have nothing in common, please!)

(10 points)

8) Prove the equality below for all integers $n \geq 1$. (20 points)

$$\sum_{l=1}^n \frac{1}{(2l-1)(2l+1)} = \frac{n}{2n+1}$$

9) Prove the inequality below for all integers $n \geq 2$. (20 points)

$$n! < n^n$$

Part 4: Review (20 points total)

10) Find $\{3,4,5,6,7\} - \{2,3,4,5\}$

(5 points)

11) What is the truth table for $P \Rightarrow Q$?

(5 points)

12) Prove that if x and y are both rational, then $x + y$ is rational.
(10 points)