Name___Solutions____ Linear Algebra; Quiz 2

1) Find a matrix in echelon form that is row-equivalent to $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 10 & 12 \\ 0 & 3 & 6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 10 & 12 \\ 0 & 3 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 6 & 12 \\ 0 & 3 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 6 & 12 \\ 0 & 0 & 0 \end{bmatrix}$$

2) Is the vector $\begin{bmatrix} 0 \\ 12 \\ 6 \end{bmatrix}$ in the span of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix} \right\}$? Why or why not?

First note that asking if it is in the span is the same thing as asking if it is a linear combination. Then note that because the two matrices are row equivalent, we know that:
$\begin{bmatrix} 0 \\ 12 \\ 6 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$ if and only if $\begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$.

Now to answer the question: yes, it is a linear combination of these two vectors. Using the above we can see this in three different ways.

1) Thinking about the matrix $A = [\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3] = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 6 & 12 \\ 0 & 0 & 0 \end{bmatrix}$ in $A\vec{x} = \vec{b}$ we see that $\vec{x}_3$ is a free variable, and so $\vec{a}_3$ is a linear combination of $\vec{a}_1$ and $\vec{a}_2$.

2) Thinking about the augmented matrix $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 6 & 12 \\ 0 & 0 & 0 \end{array}\right]$, this represents the system of equations:

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ 6x_2 &= 12 \\ 0 &= 0 \end{aligned}$$

which tells us that indeed the third vector is a linear combination of the first two. In particular $x_2 = 2, x_1 = -4$.

3) Thinking about the system of equations directly, we have:

$$\begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$$

which is the same as

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ 6x_2 &= 12 \end{aligned}$$

which has the solution $x_1 = -4, x_2 = 2$.