

Name Solutions _____ Linear Algebra; Quiz 4

An $n \times n$ matrix is known to have full column span, that is, its columns span the largest possible space among all $n \times n$ matrices. In a numbered list state everything you can about this matrix and/or anything that is related.

(For full credit you should have at least 5 “interesting” facts. Each additional fact will be given additional, but diminishing, credit.)

The following is an incomplete list of equivalent statements:

1. The columns of the matrix span \mathbb{R}^n .
2. The columns of the matrix are linearly independent.
3. In echelon form the matrix is upper triangular.
4. As a matrix equation every variable is a leading variable.
5. The associated linear transformation is onto.
6. The associated linear transformation is one-to-one.
7. The matrix is invertible.
8. The associated homogenous equation has exactly one solution.
9. Any associated nonhomogenous equation has at most one solution.

Note that the equivalence of many of these statements hinge on the fact that the matrix is square.