Suppose the linear operator $T: \mathbb{R}^4 \to \mathbb{R}^5$ is one-to-one. What are three equivalent statements?

There are many equivalent statements, here is a selection of one from each type of object:

The kernel of $T$ is trivial. That is, $\ker(T) = \{0\}$.

The columns of $[T]$ are linearly independent.

The system of equations $[T]\vec{x} = \vec{0}$ has only the zero solution.

Which of the following are bases for $\mathbb{R}^3$? Circle those that are.

\[
\begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
1 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\]

\[
\text{span}\left(\begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}\right)
\]