Name___Solutions____ Linear Algebra; Quiz 7

Suppose the linear operator $T: \mathbb{R}^4 \to \mathbb{R}^5$ is one-to-one. What are three equivalent statements?

There are many equivalent statements, here is a selection of one from each type of object:

The kernel of T is trivial. That is, $\ker(T) = \{\vec{0}\}$.

The columns of $[T]$ are linearly independent.

The system of equations $[T]\vec{x} = \vec{0}$ has only the zero solution.

Which of the following are bases for $\mathbb{R}^3$? Circle those that are.

$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$$

$$\boxed{\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}}$$

$$span\left(\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}\right)$$

$$\left\{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\right\}$$