Name____ Linear Algebra; Quiz 8

Consider the linear operator $T: \mathbb{R}^5 \to \mathbb{R}^4$ given by its associated matrix below.

$$[T] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1) What is the dimension of the range of T?

$$\dim(\text{range}(T)) = 3$$

2) What is the range of T?

$$\text{range}(T) = \text{span}\left(\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\2\\0\end{bmatrix}\right\}\right) = \text{span}\left(\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\2\\0\end{bmatrix}\right\}\right)$$

3) Is $\begin{bmatrix} 23958798347982982024 5\pi \\ 238723893684942390560 9 \\ 3495489349804509.34 \\ 0 \end{bmatrix}$ in the range of T?

Yes, as seen from the basis we can choose whatever we like for the first three coordinates. But the last coordinate must be zero.

4) What is the dimension of the kernel of T?

$$\dim(ker(T)) = 2$$

5) What is the kernel of T?

$$\text{span}\left(\left\{\begin{bmatrix}1\\-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}\right\}\right)$$