Name____Solutions_____ Linear Algebra; Quiz 10

Consider the matrix given below.

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

1) Find a basis for the row space of $[T]$.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^t, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^t, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^t \right\}$$

2) Find a basis for the range of T.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

3) Find a basis for the null space of $[T]$

This question doesn't make any sense. The null space is $\{\vec{0}\}$ which is the unique space without a basis.

4) What is the associated linear operator T?

$$T: \mathbb{R}^3 \to \mathbb{R}^4$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_1 + x_2 \\ x_2 + x_3 \\ x_3 \end{bmatrix}$$

5) Let $[T]^t$ denote the transpose of the matrix $[T]$. Find $[T]^t \cdot [T]$.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$