Name__Solutions____ Linear Algebra; Quiz 12

Consider the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

1) Find all the eigenvalues of A. Circle your answer.

2) Find all the eigenspaces of A. Box your answer(s).

$$\begin{vmatrix} 3-x & 1 & 2 \\ -1 & 1-x & 2 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)\cdot[(3-x)(1-x)+1] = (1-x)(x^2-4x+4) = (1-x)(x-2)^2$$

$$\lambda_1 = 1; \lambda_2, \lambda_3 = 2$$

$\lambda_1 = 1$:

$$\begin{bmatrix} 3-1 & 1 & 2 \\ -1 & 1-1 & 2 \\ 0 & 0 & 1-1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -6 \\ 1 \end{bmatrix}$$

$\lambda_2, \lambda_3 = 2$:

$$\begin{bmatrix} 3-2 & 1 & 2 \\ -1 & 1-2 & 2 \\ 0 & 0 & 1-2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

The eigenspaces are then:

$\text{span}(\vec{v}_1)$ and $\text{span}(\vec{v}_2)$.

3) Find all the eigenvalues of the matrix below:

$$\begin{bmatrix} 1 & x & y & \pi \\ 0 & 2 & 4! & 6.2 \\ 0 & 0 & 3 & 2i \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$1, 2, 3, 4$

4) Suppose A is 5×5 matrix with 5 different eigenvalues. How many nontrivial eigenspaces does A have?

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