Name_____Solutions_____ Linear Algebra; Quiz 13

Choose and complete one of the following problems:

1) Let $\beta_1 = \left\{\begin{bmatrix}2\\4\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}\right\}, \beta_2 = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$. Write the vector $\begin{bmatrix}1\\3\end{bmatrix}_{\beta_1}$ in terms of β_2.

2) Diagonalize the matrix $\begin{bmatrix}2&1\\4&5\end{bmatrix}$. Express your answer as an equation involving the matrix and its diagonalization.

1) $[I_2]^S_{\beta_1} = \begin{bmatrix}2&2\\4&5\end{bmatrix}$. $[I_2]^{\beta_2}_S = \begin{bmatrix}1&1\\0&2\end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix}2&-1\\0&1\end{bmatrix}$

$\begin{bmatrix}1\\3\end{bmatrix}_{\beta_1} = \frac{1}{2}\begin{bmatrix}2&-1\\0&1\end{bmatrix}\begin{bmatrix}2&2\\4&5\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}2&-1\\0&1\end{bmatrix}\begin{bmatrix}7\\19\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-5\\19\end{bmatrix}_{\beta_2}$

2)

$$(2-x)(5-x) - 4 = 0$$
$$x^2 - 7x + 6 = 0$$
$$(x-6)(x-1) = 0$$

$$\lambda_1 = 6; \lambda_2 = 1$$

$$\begin{bmatrix}2-6&1\\4&5-6\end{bmatrix} \sim \begin{bmatrix}-4&1\\4&-1\end{bmatrix} \sim \begin{bmatrix}-4&1\\0&0\end{bmatrix}$$
$$\vec{v}_1 = \begin{bmatrix}1\\4\end{bmatrix}$$

$$\begin{bmatrix}2-1&1\\4&5-1\end{bmatrix} \sim \begin{bmatrix}1&1\\4&4\end{bmatrix} \sim \begin{bmatrix}1&1\\0&0\end{bmatrix}$$
$$\vec{v}_1 = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$\begin{bmatrix}2&1\\4&5\end{bmatrix} = \begin{bmatrix}1&1\\4&-1\end{bmatrix}^{-1}\begin{bmatrix}6&0\\0&1\end{bmatrix}\begin{bmatrix}1&1\\4&-1\end{bmatrix}$$
$$= \frac{-1}{5}\begin{bmatrix}-1&-1\\-4&1\end{bmatrix}\begin{bmatrix}6&0\\0&1\end{bmatrix}\begin{bmatrix}1&1\\4&-1\end{bmatrix}$$