Consider the following two matrices:

\[
A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix};
B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}
\]

1) Describe the columnspace of each matrix, that is, the span of the columns of the matrix.

2) Consider the associated linear transformations \( T_A \) and \( T_B \). Describe the domain, codomain, and range of these functions.

3) Describe \( T_A \) and \( T_B \) as functions: that is, if the input is, say, \( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \) then what is the output?

4) Let \( T_1 = T_A \circ T_B \). Find \([T_1]\), the matrix associated to \( T_1 \). (Hint: use matrix multiplication. Use the domain and codomain of these functions to determine in which order to multiply)

5) Describe \( T_1 \) as a function, as well as its domain, codomain, and range.

6) Let \( T_2 = T_B \circ T_A \). Find \([T_2]\), the matrix associated to \( T_2 \).

7) Describe \( T_2 \) as a function, as well as its domain, codomain, and range.

8) Are any of \( T_A, T_B, T_1, T_2 \) one-to-one and/or onto?

9) Which of \( T_A, T_B, T_1, T_2 \) are invertible?

10) For the \( T_i \) that are invertible, give its domain, codomain, and range.

11) For the \( T_i \) that are invertible, find \([T_i]\).

12) For the \( T_i \) that are invertible, verify that \([T_i] \cdot [T_i]^{-1} = I_n\) and that \([T_i]^{-1} \cdot [T_i] = I_n\).

13) Find \( A^{-1} \cdot T_1 \cdot B \cdot T_2 \cdot A^{-1} \), if it makes sense to do so.