

1) Given the system of equations below, describe the solution set. (Don't find it, just describe it in one English sentence or mathematical expression) (3 points)

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 5 \\x_2 + x_3 + x_4 &= 2 \\x_3 + x_4 &= 7\end{aligned}$$

2) Given the system of equations below, describe the solution set. (Don't find it, just describe it in one English sentence or mathematical expression) (3 points)

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 5 \\x_2 + x_3 + x_4 &= 2\end{aligned}$$

3) Given the system of equations below, describe the solution set. (Don't find it, just describe it in one English sentence or mathematical expression) (3 points)

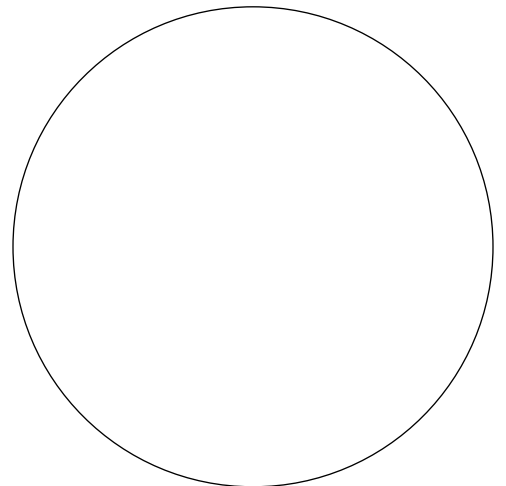
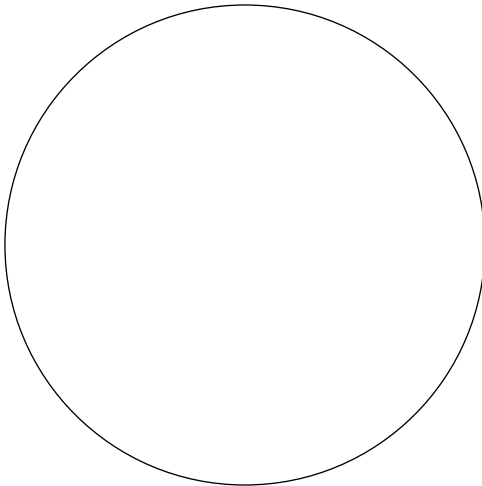
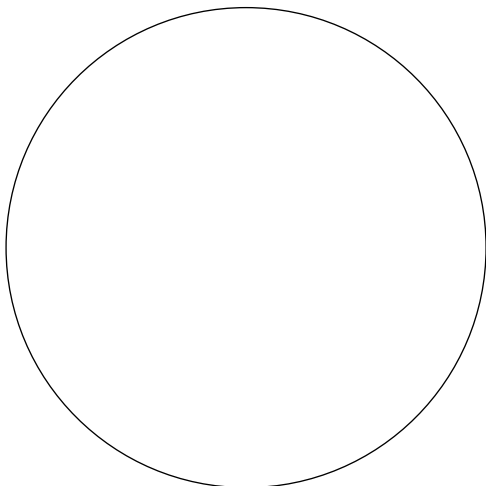
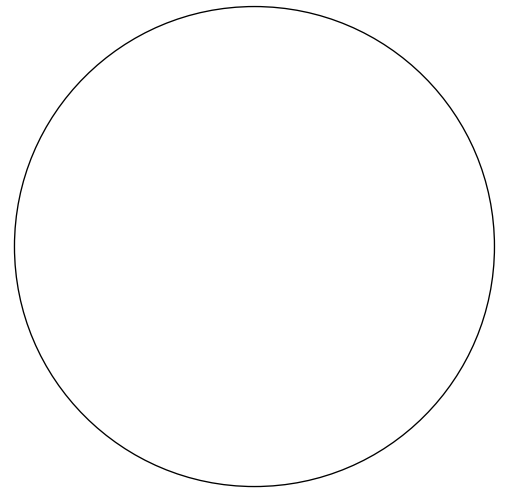
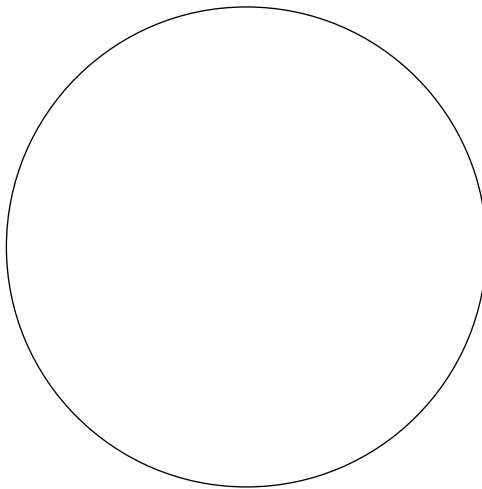
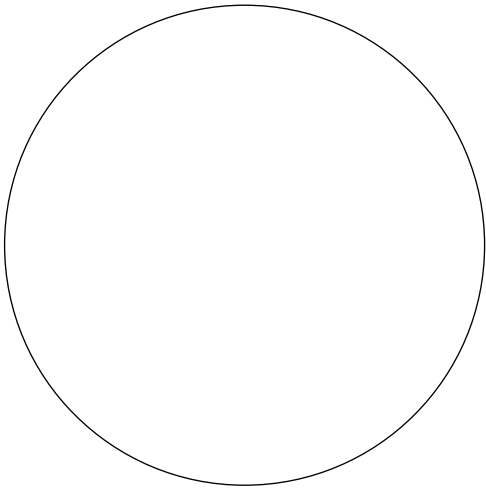
$$\begin{aligned}x_1 + x_2 + x_3 &= 5 \\x_2 + x_3 &= 2 \\x_3 &= 7\end{aligned}$$

4) Is $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ a solution to the matrix equation $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$? Why or why not? (3 points)

5) Define the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ via $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ 0 \end{bmatrix}$. Explain or show why T is a linear transformation. (8 points)

6) Let T be a linear transformation from \mathbb{R}^n to \mathbb{R}^m . Place each of the following terms in the circles below such that all items in a circle are equivalent. Note that not all circles may be used. (20 points)

- a) T is one-to-one.
- b) T is onto.
- c) The range of T is \mathbb{R}^m .
- d) The columns of T are linearly independent.
- e) The columns of T are linearly dependent.
- f) The columns of T span \mathbb{R}^m .
- g) $[T]$ has full column span.
- h) $[T]$'s echelon form shows that the corresponding equation has no free variables.
- i) $[T]\vec{x} = \vec{0}$ has exactly one solution.
- j) $[T]\vec{x} = \vec{0}$ has multiple solutions.
- k) $[T]\vec{x} = \vec{b}$ has at most one solution.
- l) $[T]\vec{x} = \vec{b}$ has multiple solutions.



7) Find a matrix in row echelon form that is row equivalent to the matrix below. (8 points)

$$\begin{bmatrix} 2 & 0 & 1 \\ 4 & 1 & 2 \\ 8 & 4 & 4 \end{bmatrix}$$

8) Find a matrix in reduced row echelon form that is row equivalent to the matrix below. (12 points)

$$\begin{bmatrix} 2 & 0 & 1 \\ 4 & 1 & 2 \\ 8 & 4 & 4 \end{bmatrix}$$

9) Find the inverse of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 9 & 0 \\ 5 & 0 & 0 & 6 \end{bmatrix}$$

10) Solve for \vec{x} . (10 points)

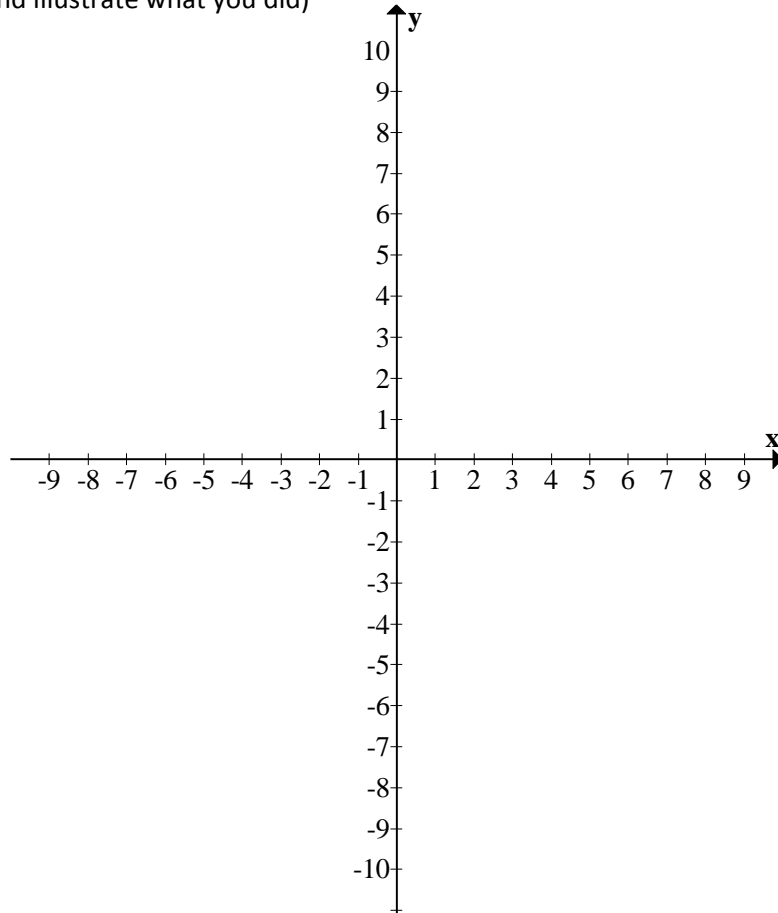
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 9 & 0 \\ 5 & 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

11) On the graph below, sketch and label the two follow vectors below. (12 points)

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(For \vec{v}_2 , find it graphically and illustrate what you did)



12) Which of the following collections of vectors are linearly dependent? Circle them. (8 points)

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$