Name____ Linear Algebra; Test 2

1) Give a matrix with the following column space: (5 points)

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\6\\2\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}\right)$$

2) Give a linear operator whose associated matrix has the following row space: (5 points)

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\6\\2\end{bmatrix}^t,\begin{bmatrix}0\\0\\1\end{bmatrix}^t\right\}\right)$$

3) Give a matrix with the following null space: (5 points)

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\6\\2\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}\right)$$

4) Give an example of a homogeneous system of equations in which the associated linear transformation has nontrivial kernel. (5 points)

5) Let $T: \mathbb{R}^m \to \mathbb{R}^n$ be a linear operator with trivial kernel. Prove that the columns of $[T]$ are linearly independent. (10 points)

6) Find a basis for $\left\{ \begin{bmatrix} 0 \\ 1 \\ a \\ a+b \\ c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$. (5 points)

7) Give an example of a linear operator T such that the associated linear system $[T]\vec{x} = \vec{b}$ has a unique solution for all $\vec{b} \in \mathbb{R}^3$. (5 points)

8) Find the null space of $\begin{bmatrix} 1 & -5 \\ -3 & 15 \\ 2 & -10 \end{bmatrix}$. (10 points)

9) Alice is an aspiring linear algebraist. Find a set of three vectors in $\mathbb{R}^4$ such that when any one is removed, Alice can find two new vectors to add to the set to make a basis for $\mathbb{R}^4$. (5 points)

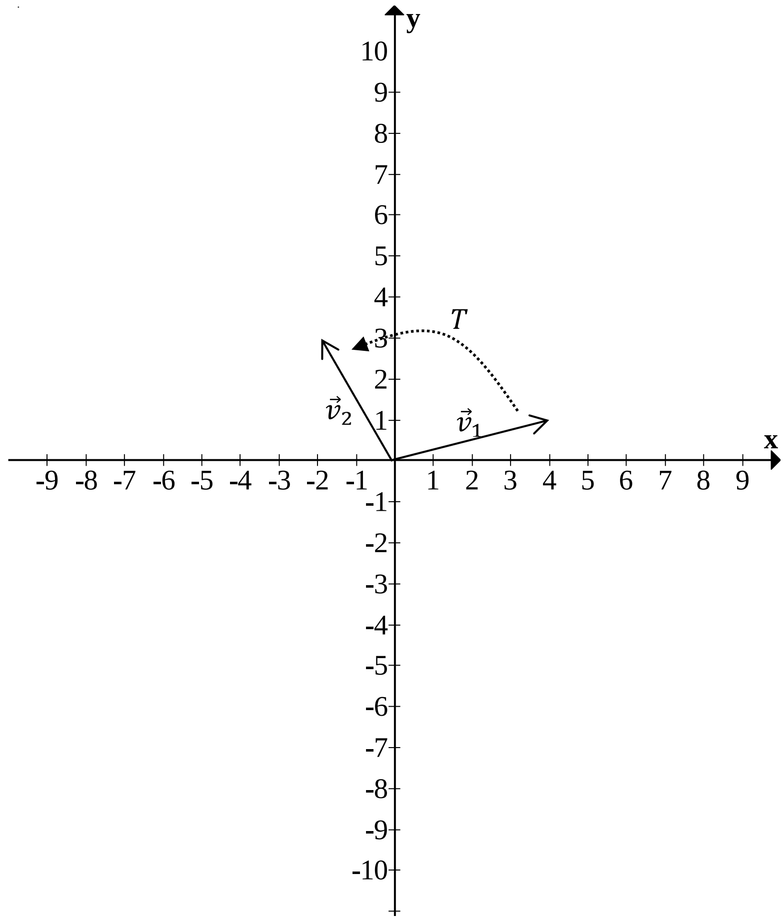
10) If $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 \end{bmatrix}$, find $[T^{-1}]$. That is, find the associated matrix of the inverse function. (5 points)

11) A 7×5 matrix has just 3 linearly dependent rows. How many free variables are there in the associated system of homogenous equations? (10 points)

12) Express 432 different bases for $\mathbb{R}^4$. (Hint: Don't try to write them all down) (5 points)

13) Find two examples of dimension 2 spaces. (5 points)

14) The graph below illustrates two vectors, $\vec{v}_1$ and $\vec{v}_2$. The picture illustrates that $T(\vec{v}_1) = \vec{v}_2$. Find the associated matrix$[T]$. (10 points)



15) Find all values of x so that $\text{rank}(A) = 2$, when $A = \begin{bmatrix} -1 & 2 & 1 \\ 3 & 1 & 11 \\ 4 & 3 & x \end{bmatrix}$. (10 points)