Name____ Linear Algebra; Test 3

Throughout the test simplify all answers except where stated otherwise.

1) Find the following: (10 points)

$$\begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1/6 & 0 & 0 & 1/2 \end{vmatrix}$$

2) Find the eigenvalues and eigenvectors of the following matrix: (10 points)

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

3) Find a matrix P and diagonal matrix D such that $A = PDP^{-1}$, where A is the matrix below. (20 points)

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $\beta_1 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 4 \\ 3 \end{bmatrix} \right\}$, and $\beta_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \right\}$.

4) Write $\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}_{\beta_1}$ in terms of the standard basis. (10 points)

5) Write $\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}_S$ in terms of β_1. (10 points)

(Do not simplify your answer: any mathematical expression that works out to the correct answer is acceptable.)

6) Find $[I_4]_{\beta_1}^{\beta_2}$, the change of basis matrix from β_1 to β_2. Be sure to show all your work. (20 points)

7) Give an example of 2×2 matrix that is not diagonalizable. (5 points)

8) Give an example of a 3×3 matrix that has determinant 42π. (5 points)

9) Suppose A is a 4×4 matrix with eigenvalues $5, 6$, and 7, with 7 having multiplicity 2. If A is not diagonalizable, what is the rank of $A - 7I_4$? (5 points. Provide an explanation for partial credit; otherwise all or nothing)

10) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$. It is known that two eigenvectors of A are $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the following: (5 points)

$$\begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^{5} \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}$$