Name ______Solutions____ Linear Algebra, Quiz 2

For problems 1-4 use the augmented matrix below.

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & 1 & 0 & 1 & | & 2 \\ 0 & 0 & 0 & 1 & 1 & | & 0 \end{bmatrix}$$

1) What is the dimension of the solution set?

2, as there are two free variables and the system is consistent.

2) Write the system as a matrix equation.

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

3) Write the system as a system of linear equations.

$$\begin{aligned} x_1 + 2x_2 \qquad\quad + x_5 &= 0 \\ x_3 \quad + x_5 &= 2 \\ x_4 + x_5 &= 0 \end{aligned}$$

4) Find the solution set.

The system is already in echelon form, so we see immediately that x_2 and x_5 are free variables. (They correspond to columns that do not have a pivot).

Then solve for the others, in reverse order: x_4, then x_3, then x_1. Place the solutions into a column vector:

$$\begin{bmatrix} -2x_2 - x_5 \\ x_2 \\ 2 - x_5 \\ -x_5 \\ x_5 \end{bmatrix}$$

Then to get the solution set we place this into a set using proper set notation:

$$\left\{ \begin{bmatrix} -2x_2 - x_5 \\ x_2 \\ 2 - x_5 \\ -x_5 \\ x_5 \end{bmatrix} : x_2, x_5 \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -1 \\ 0 \\ -1 \\ -1 \\ 1 \end{bmatrix} x_5 : x_2, x_5 \in \mathbb{R} \right\}$$

5) Row reduce the matrix below. That is, find the matrix in reduced row echelon form that is row equivalent to it.

$$\begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\qquad$ $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_2 \to \frac{1}{3} R_2$