Name _____Solutions____ Linear Algebra, Quiz 3

Let A be the matrix below on the left, its row reduced echelon form is on the right.
Let T be the linear operator associated to A.

$$\begin{bmatrix} 1 & 1 & 5 & 9 & -34 \\ 2 & 2 & 6 & 1 & -22 \\ 3 & 3 & 7 & 2 & -28 \\ 4 & 4 & 8 & 3 & -34 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1) What is the domain of T?

$\mathbb{R}^5$ (Look at the number of columns, this is where the input lives)

2) What is the codomain of T?

$\mathbb{R}^4$ (Look at the number of rows, this is where the output lives)

3) What is the range of T?

Recall how a linear operator is defined via a matrix equation; we're interested in the columnspace:

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}5\\6\\7\\8\end{bmatrix}, \begin{bmatrix}9\\1\\2\\3\end{bmatrix}, \begin{bmatrix}-34\\-22\\-28\\-34\end{bmatrix}\right\}\right) = \operatorname{span}\left(\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}5\\6\\7\\8\end{bmatrix}, \begin{bmatrix}9\\1\\2\\3\end{bmatrix}\right\}\right)$$

4) What is a basis for the range of T?

Looking at the reduced form, we see that there are pivots in the 1st, 3rd, and 4th columns:

$$\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}5\\6\\7\\8\end{bmatrix}, \begin{bmatrix}9\\1\\2\\3\end{bmatrix}\right\}$$

5) What is the kernel of T?

By solving for each variable starting from the right, we find that:

$$x_5 \in \mathbb{R}$$
$$x_4 = 2x_5$$
$$x_3 = 3x_5$$
$$x_2 \in \mathbb{R}$$
$$x_1 = -x_2 + x_5$$

We must put these into a vector and those vectors into a set to obtain the kernel:

$$\ker(T) = \left\{\begin{bmatrix}-1\\1\\0\\0\\0\end{bmatrix} x_2, \begin{bmatrix}1\\0\\3\\2\\1\end{bmatrix} x_5 : x_2, x_5 \in \mathbb{R}\right\}$$