

1) Find the following:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & \pi \\ 0 & 0 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & \pi \\ 0 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & \pi \\ 0 & 0 \\ 5 \cdot 2 + 7 & 5 \cdot \pi + 7 \cdot 3 \end{bmatrix} = \begin{bmatrix} 2 & \pi \\ 0 & 0 \\ 17 & 21 + 5\pi \end{bmatrix}$$

2) Find the following:

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 627 \\ \pi & 16 & 1 & 1 \\ 2\pi & 32 & 2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 627 \\ \pi & 16 & 1 & 1 \\ 2\pi & 32 & 2 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 0 & 627 \\ 16 & 1 & 1 \\ 32 & 2 & 3 \end{vmatrix} + 0 + 0 + 0 \\ = 2 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 0 + 627 \cdot \begin{vmatrix} 16 & 1 \\ 32 & 2 \end{vmatrix} \\ = 2 \cdot (3 - 2) + 627 \cdot 0 \\ = 2$$

3) For what values of x is the matrix below NOT invertible?

$$\begin{bmatrix} -x & 2 \\ 1 & 1-x \end{bmatrix}$$

Taking the determinant we get: $\begin{vmatrix} -x & 2 \\ 1 & 1-x \end{vmatrix} = -x \cdot (1-x) - 2 = x^2 - x - 2 = (x-2)(x+1)$

The matrix is invertible if and only if the determinant is not zero. Hence it is not invertible if and only if the determinant is zero. This occurs when $x = 2$ or $x = -1$.