Name ___Solutions____ Linear Algebra, Quiz 6

1) Find an eigenvector for the matrix below.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 2 & 4 \end{bmatrix}$$

First we find the eigenvalues by finding the determinant of $A - \lambda I_3$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 1 & 2-\lambda & 3 \\ 0 & 2 & 4-\lambda \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & 2-\lambda & 3 \\ 0 & 2 & 4-\lambda \end{vmatrix} = (1-\lambda) \cdot [(2-\lambda)(4-\lambda) - 6)]$$

I see immediately that 1 is an eigenvalue because 1 is a solution to the equation below, so I don't need to sort out the rest.

$$(1-\lambda) \cdot [(2-\lambda)(4-\lambda) - 6)]$$

Using this eigenvalue, we find the matrix $A - 1 \cdot I_3$:

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 1 & 2-1 & 3 \\ 0 & 2 & 4-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$

Anything in the null space of this matrix is an eigenvector. It's simple enough we might be able to find such a vector by inspection. But if it's not we can row reduce it:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 3 \\ 0 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Now we see the two equations below:

$$x_1 + \frac{3}{2}x_3 = 0$$

$$x_2 + \frac{3}{2}x_2 = 0$$

Noting that x_2 is a free variable, we choose $x_2 = 2$ and find the eigenvector below.

$$\vec{v} = \begin{bmatrix} -3 \\ -3 \\ 2 \end{bmatrix}$$

2) How many eigenvalues does the matrix below have? Be as thorough as possible without actually finding the eigenvalues.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$$

This matrix has 5 eigenvalues including multiplicity, because it is a 5×5 matrix. We can see this because the equation $|A - \lambda I_5| = 0$ is a 5^{th} degree polynomial in λ.

(Future reference: Soon we'll call this equation the characteristic equation and this polynomial the characteristic polynomial)