Name ____Solutions____ Linear Algebra, Quiz 7

Let A be a 9×9 matrix with 3 distinct eigenvalues.
$\lambda_1 = 5$ has only one linearly independent eigenvector $\vec{v}_1$.
$\lambda_2 = 12$ has two linearly independent eigenvectors $\vec{w}_1$ and $\vec{w}_2$.
$\lambda_3 = 27$ and no information about its eigenvectors is given.

1) What is the eigenspace corresponding to λ_1?

$$\text{span}(\vec{v}_1) = \{c\vec{v}_1 : c \in \mathbb{R}\} = \text{null}(A - 5I_9)$$

(The first answer above is the simplest and most straightforward)

2) What are possible values for the multiplicity of λ_2?

2, 3, 4, 5, 6, 7

It cannot be 1 because it has two linearly independent eigenvectors. It cannot be 8 or 9 because two roots of the characteristic polynomial are taken up by λ_1 and λ_3.

3) How many linearly independent eigenvectors can there be for λ_3?

$$1, 2, 3, 4, 5, 6$$

It cannot be 0 because λ_3 is an eigenvalue and thus has an eigenvector.
It cannot be 7, 8, or 9 because three dimensions are taken up by $\vec{v}_1, \vec{w}_1, \vec{w}_2$.

4) What are possible values for $|A|$?

$$|A| \neq 0$$

It cannot be zero because 0 is not an eigenvalue. I do wonder if we can say any more. Beyond the scope of this course: my gut feeling tells me that knowing the eigenvalues should tell us _something_ about $|A|$, but I haven't a clue what can actually be said. The problem is that $A - \lambda I_9$ involves addition/subtraction of matrices, which tends to destroy any information about the determinant of the constituent matrices.

5) What is the leading term in the characteristic polynomial of A? That is, when written in the standard order, what is the first term in $A - xI_9$?

$$-x^9$$

The variable is x because that was the variable used in $A - xI_9$.
The exponent is 9 because it is a 9×9 matrix, and the $x's$ appear on the diagonal and nowhere else.
The coefficient is -1 because all nine factors with an x look like "$(\square - x)$"