Name ____________________________________________________ Linear Algebra, Quiz 8

1) Diagonalizable the matrix below.

$$\begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix}$$

First find the eigenvalues:

$$\begin{vmatrix} 5-\lambda & 2 \\ 4 & 7-\lambda \end{vmatrix} = (5-\lambda)(7-\lambda) - 8 = \lambda^2 - 12\lambda + 27 = (\lambda-9)(\lambda-3) = 0$$

For $\lambda = 9$:

$$\begin{bmatrix} 5-9 & 2 \\ 4 & 7-9 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \sim \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$$

Solving the equation $-2x_1 + x_2 = 0$, we find that an eigenvector is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

For $\lambda = 3$:

$$\begin{bmatrix} 5-3 & 2 \\ 4 & 7-3 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Solving the equation $x_1 + x_2 = 0$, we find that an eigenvector is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Hence a diagonalization is:

$$\begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} \frac{-1}{3} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$$

2) Find a matrix that performs the two operations below simultaneously. You do not need to simplify your answer.

Stretches a vector by a factor of 3 in the direction $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Stretches a vector by a factor of 9 in the direction $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$$

This was supposed to come out to be the matrix from (1), but somehow I chose the wrong direction.