Name ____ Linear Algebra, Test 1, 9/22/2014

1) Write the vector equation below as a system of linear equations. (4 points)

$$\begin{bmatrix}2\\3\\4\end{bmatrix}x_1 + \begin{bmatrix}1\\2\\3\end{bmatrix}x_2 = \begin{bmatrix}5\\6\\7\end{bmatrix}$$

2) Determine whether or not the set below is a subspace of $\mathbb{R}^5$. If it is, find its dimension. Justify your answer. (8 points)

$$\left\{\begin{bmatrix}a\\a+b\\1+a+b+c\\0\\1\end{bmatrix} : a, b, c \in \mathbb{R}\right\}$$

3) A linear transformation is given below. Determine whether or not it is onto. Justify your answer. (8 points)

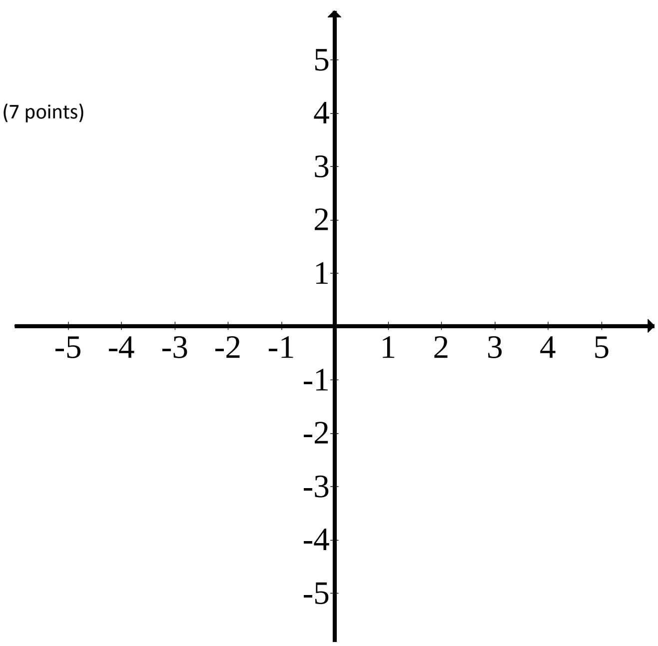
$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}2x_1 + x_2\\4x_2\\x_1 + 3x_3\\x_1 + x_2 + x_3\\6x_2 + 7x_3\end{bmatrix}$$

4) Determine which of the spaces below have dimension 2. Circle them. (6 points)

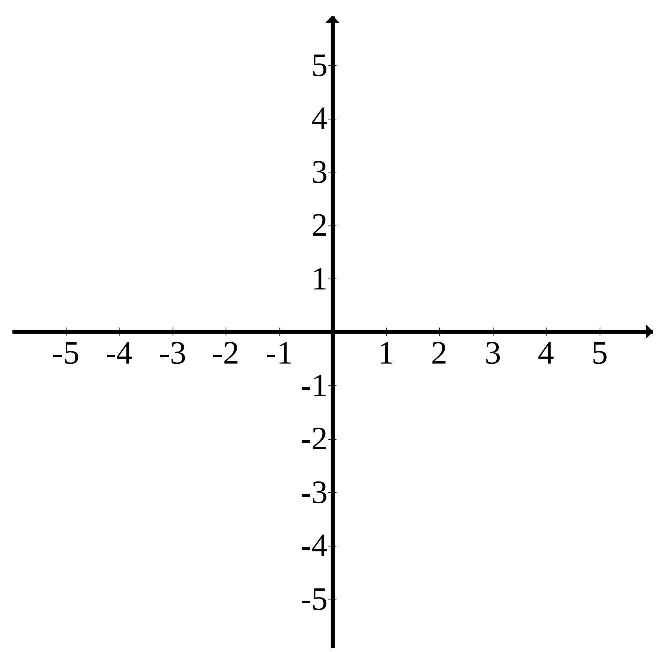
$\text{span}\left(\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}\right\}\right)$ $\qquad$ $\text{span}\left(\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\4\end{bmatrix}\right\}\right)$ $\qquad$ $\text{span}\left(\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}1\\4\end{bmatrix}\right\}\right)$

$\text{span}\left(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\4\\6\end{bmatrix}\right\}\right)$ $\qquad$ $\text{span}\left(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}\right\}\right)$ $\qquad$ $\text{span}\left(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}3\\5\\7\end{bmatrix}\right\}\right)$

5) Illustrate $\left\{\begin{bmatrix}s\\s\end{bmatrix} : s \geq 2\right\}$ on the axis below. (7 points)



6) Illustrate $\text{span}\left(\left\{\begin{bmatrix}1\\4\end{bmatrix}, \begin{bmatrix}2\\8\end{bmatrix}\right\}\right)$ on the axis below. (7 points)



7) You know that the linear transformation $T\colon \mathbb{R}^3 \rightarrow \mathbb{R}^7$ is one-to-one. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 20 points maximum)

For the problems on this page, use the basis for $\mathbb{R}^2$ below. Do not change the order of the vectors.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

8) Find the vector $\begin{bmatrix} 4 \\ 10 \end{bmatrix}$ in terms of $\mathcal{B}$. (10 points)

9) Graphically illustrate your answer to the above question. (10 points)

Use the matrix below for the problems on this page.

$$A = \begin{bmatrix} 3 & 6 & 3 & 12 \\ 1 & 2 & 2 & 7 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

10) Find the row reduced echelon form of A. (7 points)

11) Find the null space of A. (5 points)

12) A certain corporation has a vector $\vec{b}$ they are unwilling to share. But they have announced that $\vec{x} = \begin{bmatrix} 3 \\ 6.2 \\ 17 \\ 9 \end{bmatrix}$ is a solution to $A\vec{x} = \vec{b}$. Find five more solutions to $A\vec{x} = \vec{b}$. (8 points)

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