Name ____ Linear Algebra, Test 2, 11/3/2014

1) Find an equation whose solutions are the eigenvalues for the matrix below. You do not need to fully simplify the equation, but you shouldn't need any linear algebra tools to understand what the equation says. (10 points)

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 10 & -4 & 2 \end{bmatrix}$$

2) A matrix A has two identical columns. A linear transformation T is defined by $T(\vec{x}) = A\vec{x}$. What can you say about the number of solutions to the equation $T(\vec{x}) = \vec{b}$? (10 points)

Use the bases below for all the questions on this page. You do not need to simplify any answers at all.

Seriously, don't do any work just write down the answers.

$$B_1 = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\-7\end{bmatrix}\right\} \quad B_2 = \left\{\begin{bmatrix}5\\4\end{bmatrix}, \begin{bmatrix}9\\8\end{bmatrix}\right\} \quad S = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$$

3) What is $\begin{bmatrix}11\\12\end{bmatrix}_{B_1}$ in S coordinates? (5 points)

4) What is $\begin{bmatrix}13\\14\end{bmatrix}_S$ in B_2 coordinates? (5 points)

5) What is $\begin{bmatrix}15\\16\end{bmatrix}_{B_1}$ in B_2 coordinates? (5 points)

6) The linear operator T does two things simultaneously: (5 points)

- Transforms vectors from S coordinates to B_1 coordinates
- Stretches the vector by a factor of 2 in every direction.

What is the matrix associated to T?

7) The linear operator T does three things simultaneously: (3 bonus points)

- Transforms vectors from S coordinates to B_1 coordinates
- Stretches the vector by a factor of 2 in the direction $\begin{bmatrix}1\\0\end{bmatrix}$
- Stretches the vector by a factor of 3 in the direction $\begin{bmatrix}1\\2\end{bmatrix}$

What is the matrix associated to T?

8) Which of the matrices below are invertible? Circle them. (2 points each)

$$\begin{bmatrix} 17 & 23 \\ 234 & 16 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 20 & 30 \\ 0 & 16 & -23 & 4 \\ 0 & 0 & 13 & -2 \\ 0 & 0 & 0 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & -7 \\ 0 & 1 & 0 & 6 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 4 & 3 & 0 & 1 & 18 \\ -2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

9) Find the inverse of the matrix below. (12 points)

$$\begin{bmatrix} 3 & 6 & 3 \\ 1 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

10) A 5×5 matrix has determinant 3. What can you say about its null space? (10 points)

11) Using the basis $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$, graphically illustrate the sum $\begin{bmatrix} 3 \\ 0 \end{bmatrix}_B + \begin{bmatrix} 0 \\ 2 \end{bmatrix}_B$. (10 points)

12) Let $T: \mathbb{R}^6 \to \mathbb{R}^7$ and $S: \mathbb{R}^7 \to \mathbb{R}^{12}$ be two linear operators. It is known that the equation $S(T(\vec{x})) = \vec{b}$ can be solved uniquely, but another equation $S(\vec{x}) = \vec{b}_2$ has multiple solutions. Expand on this and explain as precisely as possible exactly how many solutions the equation $S(\vec{x}) = \vec{b}_2$ has. (3 bonus points)

13) Find all the eigenvectors for the matrix below. You may include $\vec{0}$ even though we don't consider it an eigenvector if it makes your answer(s) easier to write down. (10 points)

$$\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}$$

14) How would you explain the number of eigenvectors in the previous problem in terms of dimension? (3 bonus points)

15) Find $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$ (5 points)

16) A 7×7 matrix A has determinant 4. Find the determinant of A^{-1}. (5 points)

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