Name ____ Linear Algebra, Test 3, 12/1/2014

1) Diagonalize the matrix below. (20 points)

$$\begin{bmatrix} 6 & 4 \\ 2 & -1 \end{bmatrix}$$

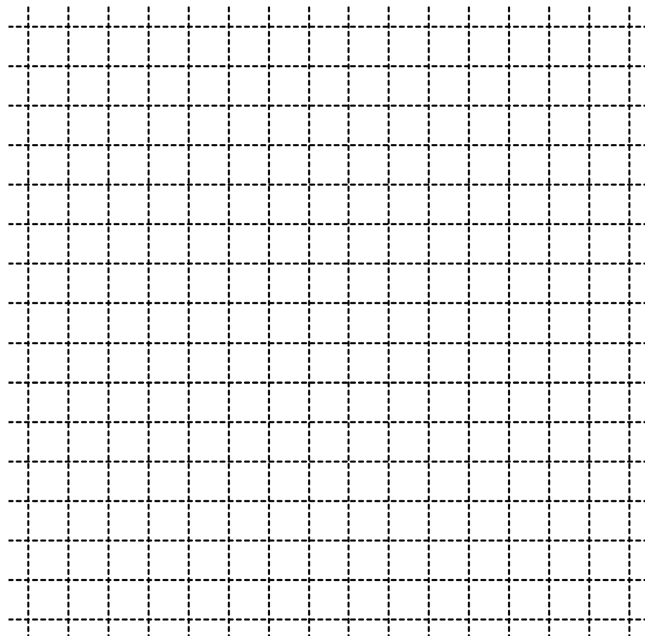
2) Alice is wandering down a red concrete road, where she happens upon the following vectors:

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}3\\1\\2\\0\end{bmatrix}, \text{ and then} \begin{bmatrix}9\\2\\2\\1\end{bmatrix}$$

“Ah-ha!” She exclaimed, “These vectors are linearly independent.” She continued down the road until she reached the gates of Sapphire Suburbia. Much to her chagrin, Alice discovered that they would not let her in if she carried vectors that were not orthogonal to each other. Can we aid Alice by orthogonalizing her vectors? Yes! Find three mutually orthogonal vectors that encode the same information as Alice’s three vectors. (20 points)

3) On the space provided, illustrate both of the following:

- The space $\operatorname{span}\left(\left\{\begin{bmatrix}7\\3\end{bmatrix}\right\}\right)$ (2 points)
- The orthogonal complement of the space in the previous bullet point. (8 points)



4) Consider the diagonalization given below and the basis $B = \left\{\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$. Note the order on the basis.

$$\begin{bmatrix}5 & 2\\4 & 7\end{bmatrix} = \begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}9 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}1/3 & 1/3\\2/3 & -1/3\end{bmatrix}$$

Find $\begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}^{17}$. (10 points. As long as your answer clearly illustrates the main point of this question, you do not need to simplify it.)

5) Let A be a 2×2 matrix with characteristic polynomial $ax^2 + bx + c$. Also, A has two distinct eigenvalues λ_1 and λ_2. Use this information to answer the following questions.

5.a) What could a be? (3 points)

5.b) What is b? (6 points)

5.c) What is c? (6 points)

6) If $A \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 3 \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, what is $A \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$? (5 points)

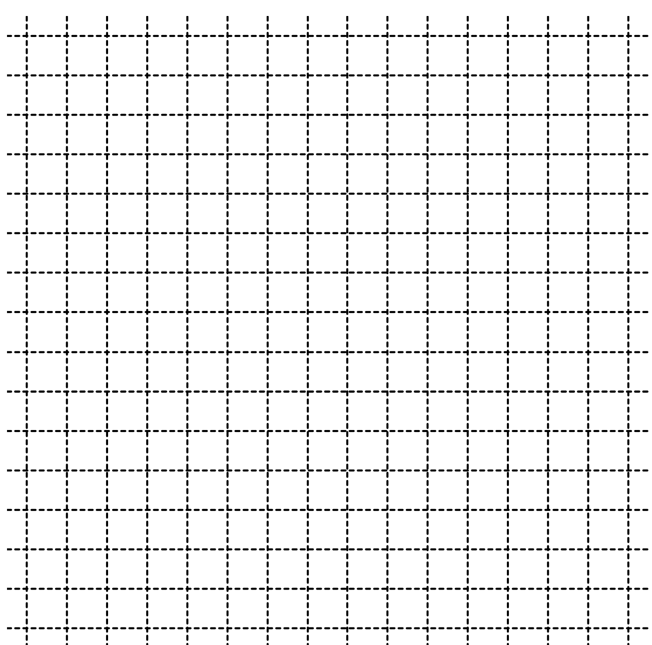
7) If $A\vec{x} = 3\vec{x}$, is it completely necessary that $\vec{x}$ is an eigenvector? Why or why not? (5 bonus points)

8) Find the length of the vector $\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$. (5 points)

9) Find the distance between the two vectors $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. (5 points)

10) Find $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 2 \\ -1 \\ 0 \\ 1 \\ 0.5 \end{bmatrix}$. (5 points)

11) On the palette below, illustrate the norm of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. (5 points)



(This page intentionally left blank. Work here will not be counted unless explicitly referenced earlier)