Name _____Solutions_____Linear Algebra, Quiz 2

1) Write the following system of linear equations as a matrix equation.

$$\begin{aligned} 2x_1 + 4x_2 - 3x_3 &= 7 \\ 5x_2 + x_3 &= 8 \\ 12x_1 + 6x_2 - 9x_3 &= 17 \end{aligned}$$

$$\begin{bmatrix} 2 & 4 & -3 \\ 0 & 5 & 1 \\ 12 & 6 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 17 \end{bmatrix}$$

2) What are the solutions to the system represented by the augmented matrix below?

$$\left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 2 & 4 & 10 \end{array}\right]$$

$$\begin{aligned} x_1 &= 7 - 2x_3 \\ x_2 &= \frac{10 - 4x_3}{2} = 5 - 2x_3 \\ x_3 &\in \mathbb{R} \end{aligned}$$

OR

$$\{(7 - 2s, 5 - 2s, s) : s \in \mathbb{R}\}$$

3) What are the leading variables in the system given in (2)?

x_1 and x_2

4) What are the free variables in the system given in (2)?

x_3

5) Reduce the matrix below to reduced row echelon form.

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 7 \\ 2 & 4 & 0 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 7 \\ 2 & 4 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -2 \\ 2 & 4 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & -6 \end{bmatrix} \xrightarrow{R_2 \to -\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -6 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 6R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 - 3R_2} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$