Name __Solutions________________________________________________Linear Algebra, Quiz 4

1) Find the following product:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & \pi & x \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & \pi & x \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6+4 \\ x & 3\pi & x \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ x & 3\pi & x \\ 0 & 0 & 0 \end{bmatrix}$$

2) Find the determinant of the matrix below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 1 \end{vmatrix} = 1 \cdot 1 \cdot \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 1 \end{vmatrix}$$

$$= 1 \cdot 1 \cdot 2 \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{vmatrix} = 1 \cdot 1 \cdot 2 \cdot 1 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{vmatrix} = 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot (1 \cdot 1 - 3 \cdot 1) = -4$$

In each case we chose to expand on the blue row or column.

3) Solve the matrix equation below for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. *You do not need to simplify your answer.*

$$\begin{bmatrix} 71 & 70 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

First note that $\begin{bmatrix} 71 & 70 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -70 \\ -1 & 71 \end{bmatrix}$ by using the formula. (The determinant is 1).

Thus we apply the multiply by this inverse matrix on both sides of the equation:

$$\begin{bmatrix} 1 & -70 \\ -1 & 71 \end{bmatrix} \begin{bmatrix} 71 & 70 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -70 \\ -1 & 71 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

This gives:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -70 \\ -1 & 71 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1 - 70b_2 \\ -b_1 + 71b_2 \end{bmatrix}$$