Name __Solutions____Linear Algebra, Quiz 5

1) In an "LU factorization", how would you describe "L" and "U"?

L is a lower triangular matrix and U is an upper triangular matrix such that A=LU.

2) Why would anyone want to use an "LU factorization"?

An LU factorization can be used to solve the system $A\vec{x} = \vec{b}$, by solving $L\vec{y} = \vec{b}$ and then $U\vec{x} = \vec{y}$.

While finding L and U takes a bit of work, each of these systems are very Easy to solve because they are triangular. Hence if this system needs to be solved several times with different right hand side vectors $\vec{b}$, then this is much more efficient than starting back at $A\vec{x} = \vec{b}$ every time.

3) Describe the "null space" of a matrix.

This is the set of all vectors which map to zero: $\{\vec{x}: A\vec{x} = \vec{0}\}$

4) Find a space S in which the projection of $\begin{bmatrix}1\\2\\3\end{bmatrix}$ onto S is $\begin{bmatrix}1\\0\\3\end{bmatrix}$.

If we think about $\mathbb{R}^3$ under the standard basis with axis labels x, y, and z, then we're looking for the xz plane:

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}\right)$$