Name _____Solutions____Linear Algebra, Quiz 6

Let $B_1 = \left\{\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}7\\0\end{bmatrix}\right\}$ and $B_2 = \left\{\begin{bmatrix}-2\\0\end{bmatrix}, \begin{bmatrix}0\\-2\end{bmatrix}\right\}$.

1) Consider the vector $\vec{x} \in \mathbb{R}^2$. In terms of basis B_1 we know $\vec{x}_{B_1} = \begin{bmatrix}1\\2\end{bmatrix}_{B_1}$. What is $\vec{x}_{B_2}$, that is, $\vec{x}$ in terms of B_2? *No need to simplify your answer.*

$$\vec{x}_{B_2} = [I_2]_{B_1}^{B_2}\vec{x}_{B_1} = [I_2]_{B_1}^{B_2}\begin{bmatrix}1\\2\end{bmatrix}_{B_1} = [I_2]_S^{B_2}[I_2]_{B_1}^S\begin{bmatrix}1\\2\end{bmatrix}_{B_1} = \left([I_2]_{B_2}^S\right)^{-1}[I_2]_{B_1}^S\begin{bmatrix}1\\2\end{bmatrix}_{B_1} = \begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}^{-1}\begin{bmatrix}1 & 7\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}_{B_1}$$

It wasn't necessary, but if you worked this out, it is:

$$\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}^{-1}\begin{bmatrix}1 & 7\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}_{B_1} = \frac{1}{4}\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}\begin{bmatrix}1 & 7\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}_{B_1} = \frac{1}{4}\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}\begin{bmatrix}15\\-1\end{bmatrix}_S = \frac{1}{4}\begin{bmatrix}-30\\2\end{bmatrix}_{B_2} = \begin{bmatrix}-7.5\\0.5\end{bmatrix}_{B_2}$$

2) Let V be the solution set to a certain system of homogenous linear equations with 7 variables. It is known that the dimension of V is 3. Associated to V is a linear operator mapping $\mathbb{R}^7$ to $\mathbb{R}^9$. This linear operator obtains as its output the right hand side of the system, regardless of whether it is zero or not. What is the dimension of the range of this linear operator? *Be sure to justify your answer.*

The dimension is 4, because $4 = 7 - 3$. That is, [number of columns] – [dimension of kernel] = [dimension of range]