Name ___Solutions____Linear Algebra, Quiz 7

1) Is $\begin{bmatrix}1\\1\end{bmatrix}$ an eigenvector for $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$? Justify your answer. (2 points)

Yes, in fact the eigenvalue is 3:

$$\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix} = 3\begin{bmatrix}1\\1\end{bmatrix}$$

2) $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$ has two eigenspaces. Find them. (6 points)

First I'll note that because there are two eigenspaces, we know each must have dimension 1. Then above we already found an eigenvector, so it's eigenspace is:

$$\operatorname{span}\left(\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}\right)$$

(We couldn't do this if the eigenspace potentially had dimension 2)

For the other eigenspace we need to find the other eigenvalue:

$$c_A(x) = \begin{vmatrix}1-x & 2\\2 & 1-x\end{vmatrix} = (1-x)(1-x) - 4 = x^2 - 2x - 3 = (x-3)(x+1)$$

Hence the other eigenvalue is -1, so we'll look at the matrix:

$$\begin{bmatrix}1-{}^{-}1 & 2\\2 & 1-{}^{-}1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$$

Which has null space:

$$\operatorname{span}\left(\left\{\begin{bmatrix}-1\\1\end{bmatrix}\right\}\right)$$

This is the other eigenspace.

3) Find an eigenvector for the matrix below. (Yes, just one eigenvector, any eigenvector will do! Remember $\vec{0}$ is not an eigenvector, though) (2 points)

$$\begin{bmatrix} 6 & 3 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

We see that the eigenvalues are 6, 2, and 5. I'll choose 5 and look at its corresponding matrix:

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In the null space of this matrix is the vector $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ which is an eigenvector.

Similarly, any vector in the following set is an eigenvector:

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c \end{bmatrix} : c \in \mathbb{R} \right\} \cup \left\{ \begin{bmatrix} 0 \\ 0 \\ c_1 \\ c_2 \\ 0 \end{bmatrix} : c_1, c_2 \in \mathbb{R} \right\} \cup \left\{ \begin{bmatrix} c \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} : c \in \mathbb{R} \right\}$$