Name ___Solutions____Linear Algebra, Quiz 8

1) Below are two vectors. Determine whether or not they are orthogonal. (2 points)

$$\vec{u} = \begin{bmatrix}1\\0\\3\\0\end{bmatrix} \quad \vec{v} = \begin{bmatrix}-3\\2\\1\\-4\end{bmatrix}$$

Yes these are orthogonal because their dot product is zero:

$$(1)(-3) + 0 + (3)(1) + 0 = -3 + 3 = 0$$

2) Find $\begin{bmatrix}0 & 0 & 1\\0 & 1 & 2\\0 & 0 & -1\end{bmatrix}^{50}$. (8 points)

Eigenvalues: 2 points
Eigenvectors: 2 points
Diagonalization: 2 points
Simplification: 2 points

$\lambda = 0$: $\begin{bmatrix}0 & 0 & 1\\0 & 1 & 2\\0 & 0 & -1\end{bmatrix} \sim \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$. So $\vec{v}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$.

$\lambda = 1$: $\begin{bmatrix}0-1 & 0 & 1\\0 & 1-1 & 2\\0 & 0 & -1-1\end{bmatrix} = \begin{bmatrix}-1 & 0 & 1\\0 & 0 & 2\\0 & 0 & -2\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$. So $\vec{v}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$.

$\lambda = -1$: $\begin{bmatrix}0+1 & 0 & 1\\0 & 1+1 & 2\\0 & 0 & -1+1\end{bmatrix} = \begin{bmatrix}1 & 0 & 1\\0 & 2 & 2\\0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}$. So $\vec{v}_3 = \begin{bmatrix}-1\\-1\\1\end{bmatrix}$.

$$\begin{bmatrix}0 & 0 & 1\\0 & 1 & 2\\0 & 0 & -1\end{bmatrix}^{50} = \begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}^{50}\begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}^{-1}$$

$$= \begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}^{50}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

Here the center matrix is an elementary matrix that performs the operation $R_1 \to 0 \cdot R_1$ to the matrix to the right of it.

$$= \begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

Here we broke the matrix on the left up to two elementary matrices.

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

Apply an elementary matrix to the matrix on the right. $R_1 \to R_1 - R_3$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & -1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

Apply an elementary matrix to the matrix on the right. $R_2 \to R_2 - R_3$

$$= \begin{bmatrix}0 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$