

Below is a basis. Find an orthogonal basis that spans the same space.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

We'll take \vec{v}_1 as the first vector: $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

As \vec{v}_2 we'll start with $\begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}$, but we want only the information that is orthogonal to \vec{v}_1 :

$$\begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Then for \vec{v}_3 we'll start with $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, but we want only the information that is orthogonal to both \vec{v}_1 and \vec{v}_2 :

$$\begin{aligned} & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1+1}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{-1+2+1}{1+4+1} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -2/3 \\ -1/3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ -1/3 \\ 1 \end{bmatrix} \end{aligned}$$

Hence the orthogonal basis is:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 1/3 \\ -1/3 \\ 1 \end{bmatrix} \right\}$$