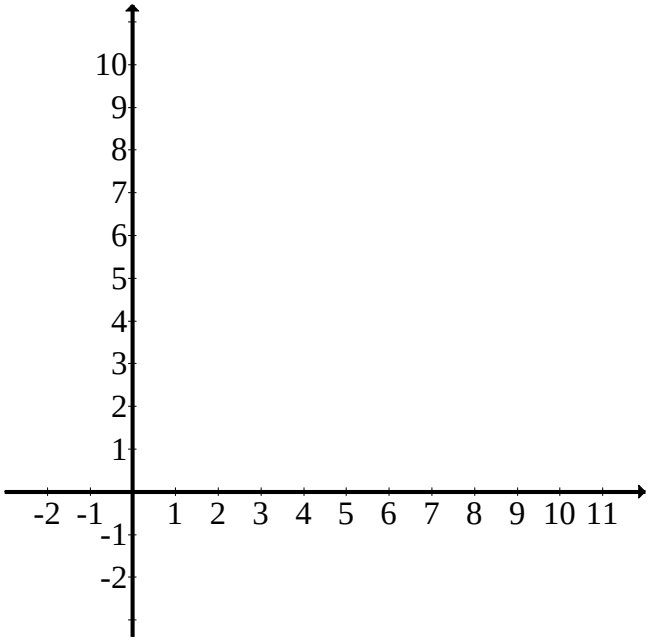
Name _____ Linear Algebra, Test 1

1) Why is the set below not a subspace of $\mathbb{R}^4$? (7 points)

$$\left\{\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} : a, b, c \in \mathbb{R}, c \geq 0\right\}$$

2) On the axis below, graph $\text{span}(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$ where $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$. (13 points)



3) Calculate the following: (5 points)

$$\begin{bmatrix}1\\2\\0\\0\\3\end{bmatrix} + 7\begin{bmatrix}0\\0\\2\\0\\1\end{bmatrix}$$

4) Find the null space of the following matrix. (10 points)

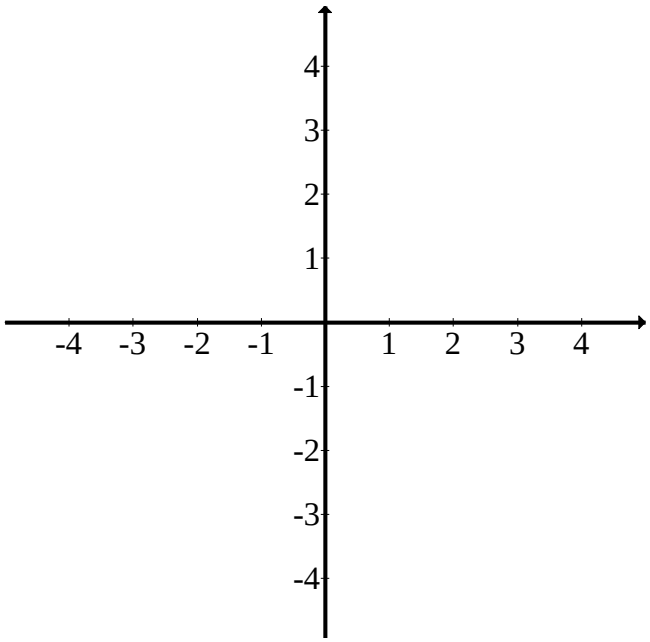
$$\begin{bmatrix}1 & 0 & 0 & 2\\0 & 0 & 1 & 9\\0 & 0 & 0 & 0\end{bmatrix}$$

5) Write the following matrix equation as a system of linear equations. (5 points)

$$\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}7\\8\\9\end{bmatrix}$$

6) On the plane below, graphically illustrate the solution to the following system: (10 points)

$$y - x = 0$$
$$y - 2x = -2$$



7) Is $\begin{bmatrix}1\\2\\0\end{bmatrix}$ in the span of the three vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}$, $\begin{bmatrix}0\\1\\1\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\end{bmatrix}$? Why or why not? (10 points)

8) Given the matrix equation below, identify which variables are free and which variables are leading. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

9) Which of the following are bases for $\mathbb{R}^2$? Circle them. (4 points)

$$\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}\right\} \qquad \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\} \qquad \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix}\right\} \qquad \left\{\begin{bmatrix} a \\ b \end{bmatrix} : a, b \in \mathbb{R}\right\}$$

10) Is the following set of vectors linearly dependent or linearly independent? Why? (11 points)

$$\left\{\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}\right\}$$

11) Reduce the following matrix to row reduced echelon form. (20 points)

$$\begin{bmatrix} 2 & 6 & 0 & 4 & 0 \\ 1 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 8 & 0 & 15 & 363 & 20 \end{bmatrix}$$

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