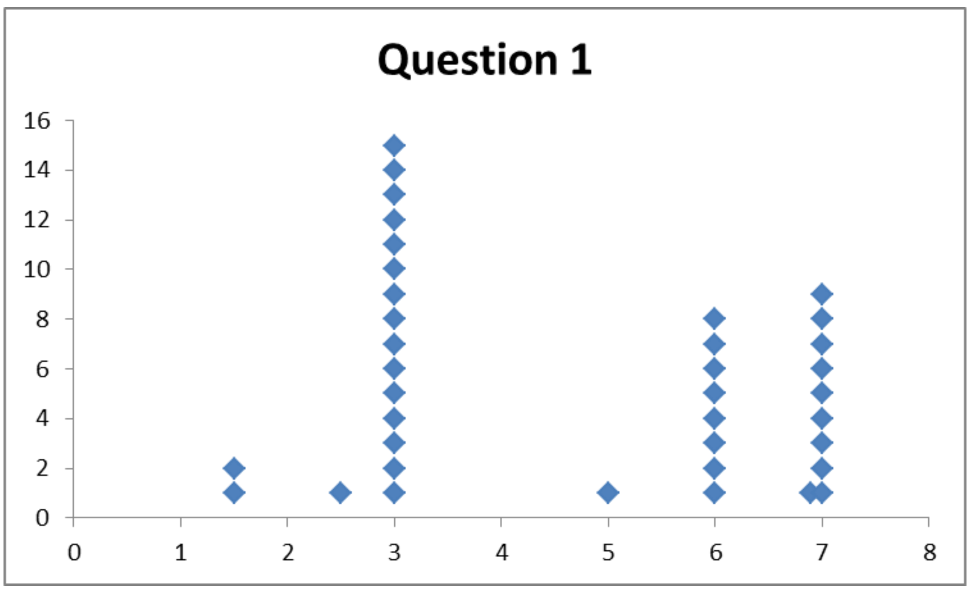
Name ___Solutions____ Linear Algebra, Test 1

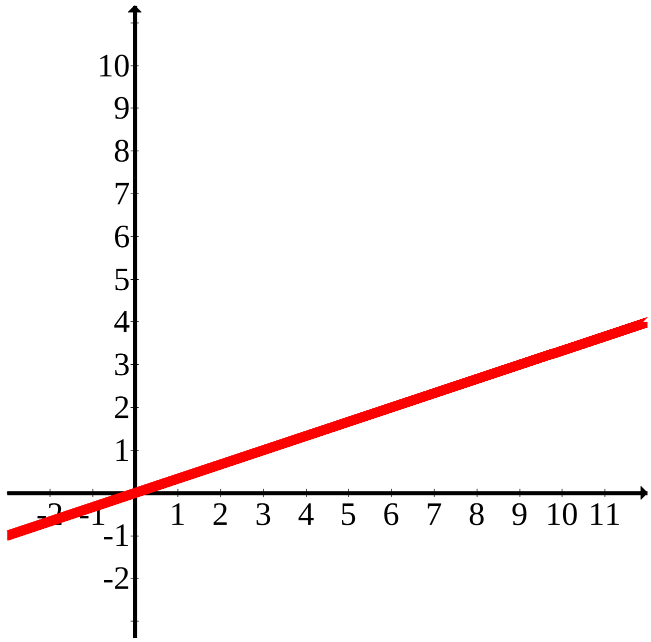
1) Why is the set below not a subspace of $\mathbb{R}^4$? (7 points)

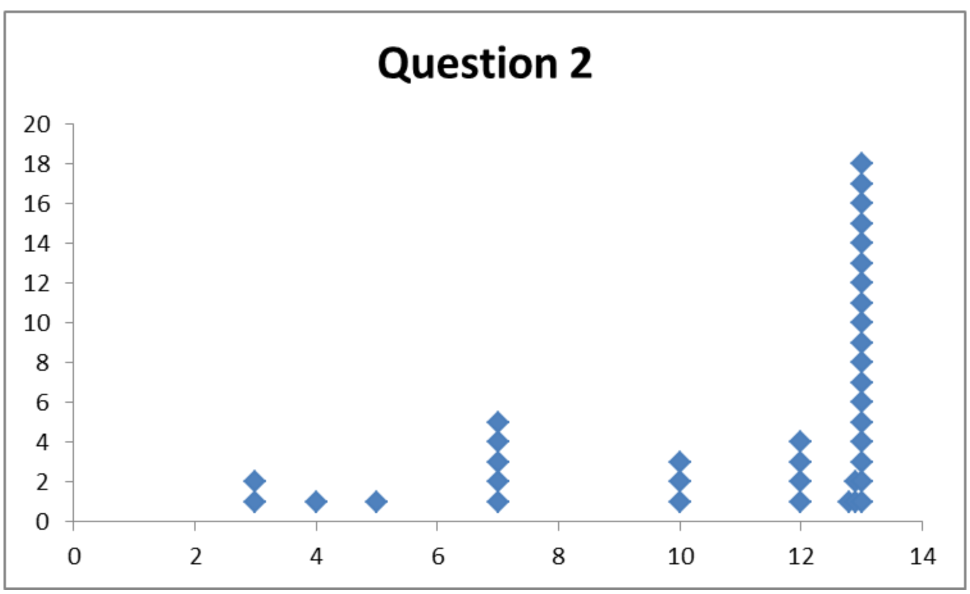
$$\left\{\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} : a, b, c \in \mathbb{R}, c \geq 0\right\}$$

It's not closed under scalar multiplication. In particular this set contains $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, but not $-1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$.



2) On the axis below, graph $\text{span}(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$ where $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$. (13 points)

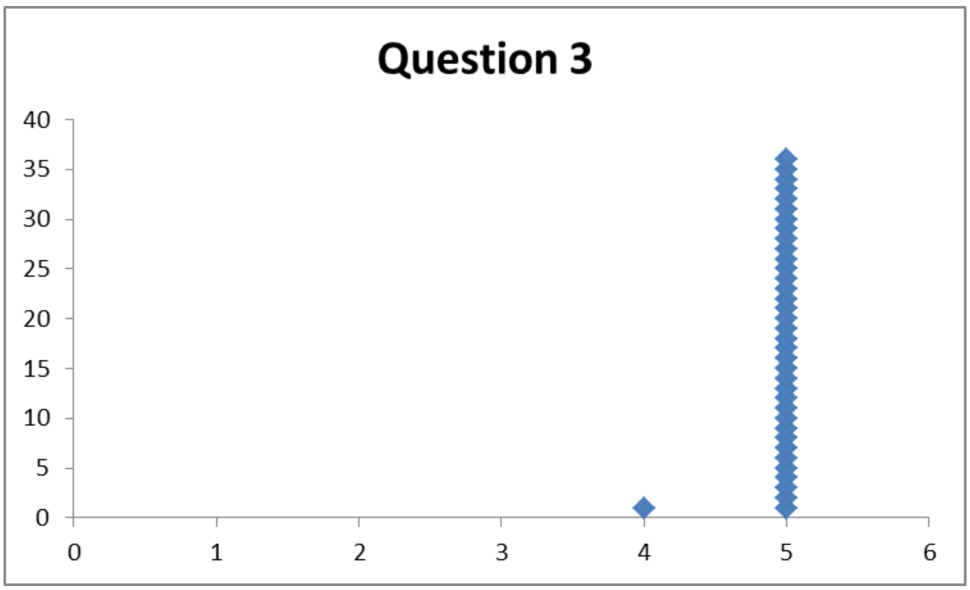




3) Calculate the following: (5 points)

$$\begin{bmatrix}1\\2\\0\\0\\3\end{bmatrix} + 7\begin{bmatrix}0\\0\\2\\0\\1\end{bmatrix}$$

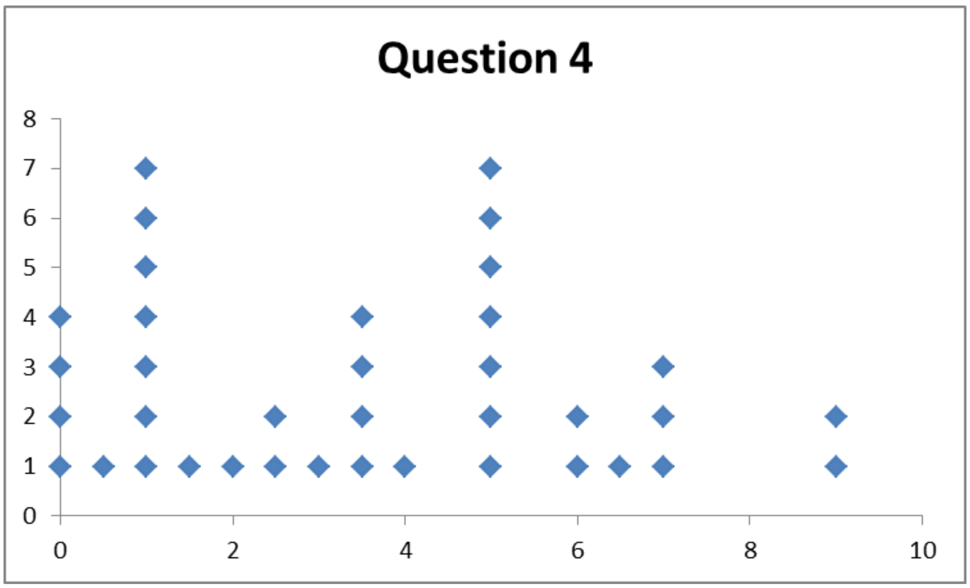
$$\begin{bmatrix}1\\2\\0\\0\\3\end{bmatrix} + 7\begin{bmatrix}0\\0\\2\\0\\1\end{bmatrix} = \begin{bmatrix}1\\2\\0\\0\\3\end{bmatrix} + \begin{bmatrix}0\\0\\14\\0\\7\end{bmatrix} = \begin{bmatrix}1\\2\\14\\0\\10\end{bmatrix}$$



4) Find the null space of the following matrix. (10 points)

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

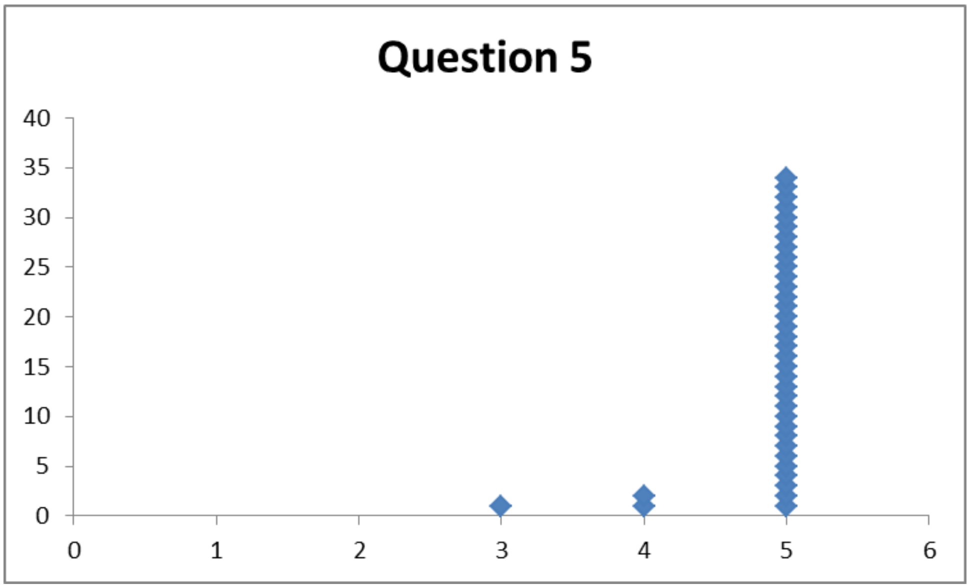
$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} -2x_4 \\ x_2 \\ -9x_4 \\ x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \left\{ s_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} -2 \\ 0 \\ -9 \\ 1 \end{bmatrix} : s_1, s_2 \in \mathbb{R} \right\}$$



5) Write the following matrix equation as a system of linear equations. (5 points)

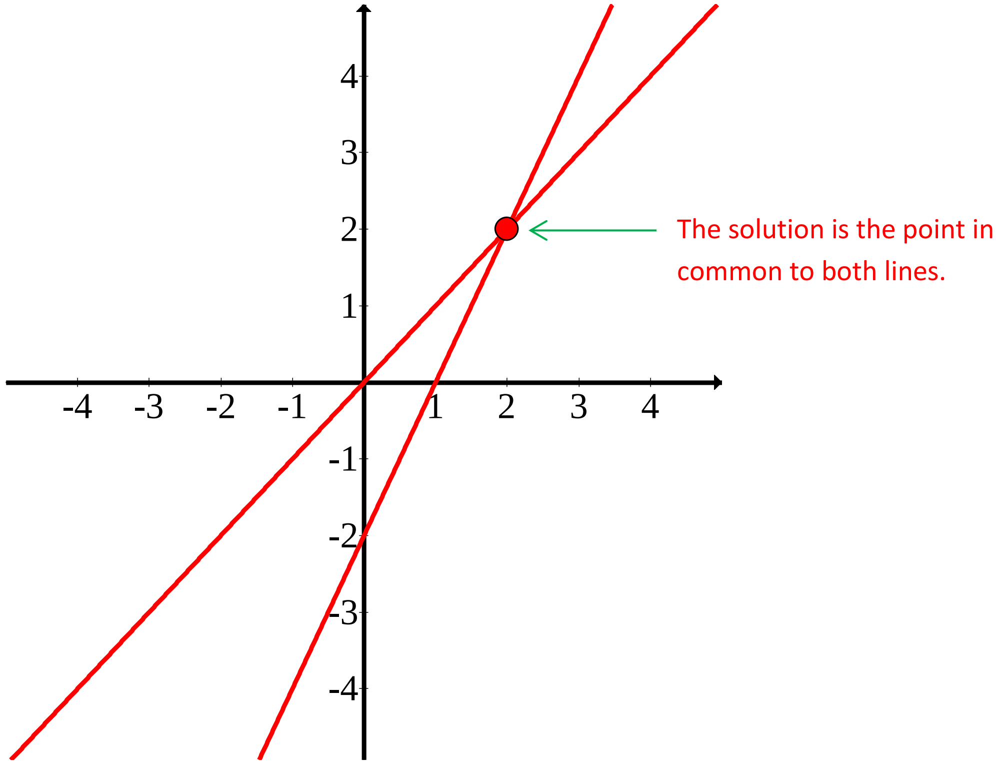
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

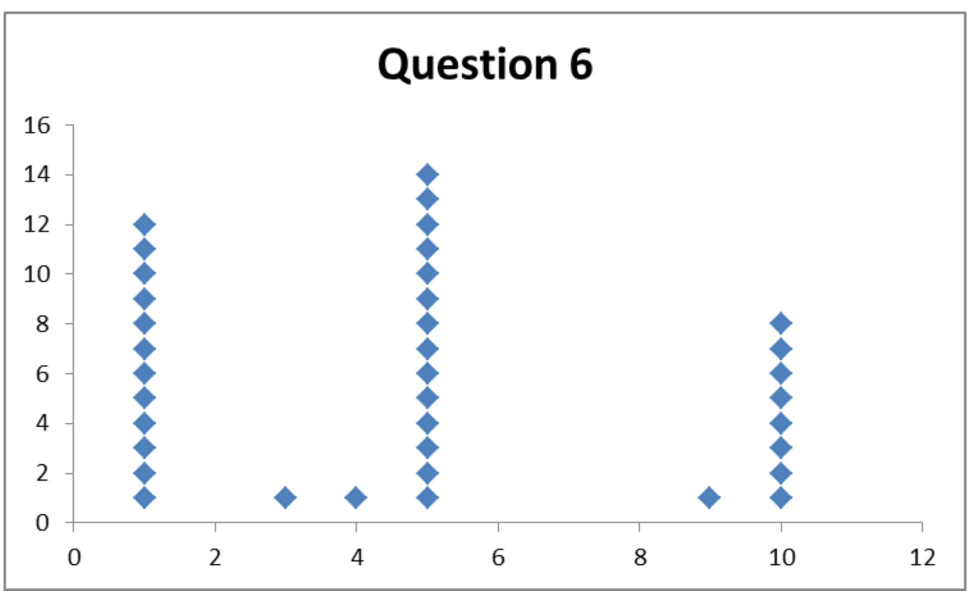
$$x + 2y = 7$$
$$3x + 4x = 8$$
$$5x + 6y = 9$$



6) On the plane below, graphically illustrate the solution to the following system: (10 points)

$$y - \ x = \ 0$$
$$y - 2x = -2$$





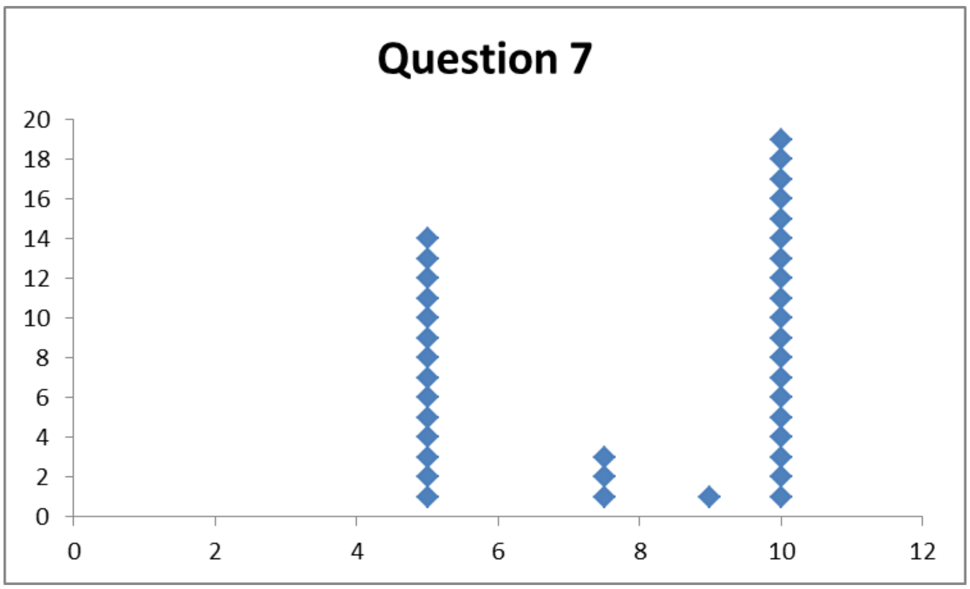
7) Is $\begin{bmatrix}1\\2\\0\end{bmatrix}$ in the span of the three vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$? Why or why not? (10 points)

Yes, as illustrated below.

$$\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix} + 2\begin{bmatrix}0\\1\\1\end{bmatrix} - 3\begin{bmatrix}0\\0\\1\end{bmatrix}$$

You could also get this by row reducing the following matrix, and the noting that the three vectors must span all of $\mathbb{R}^3$.

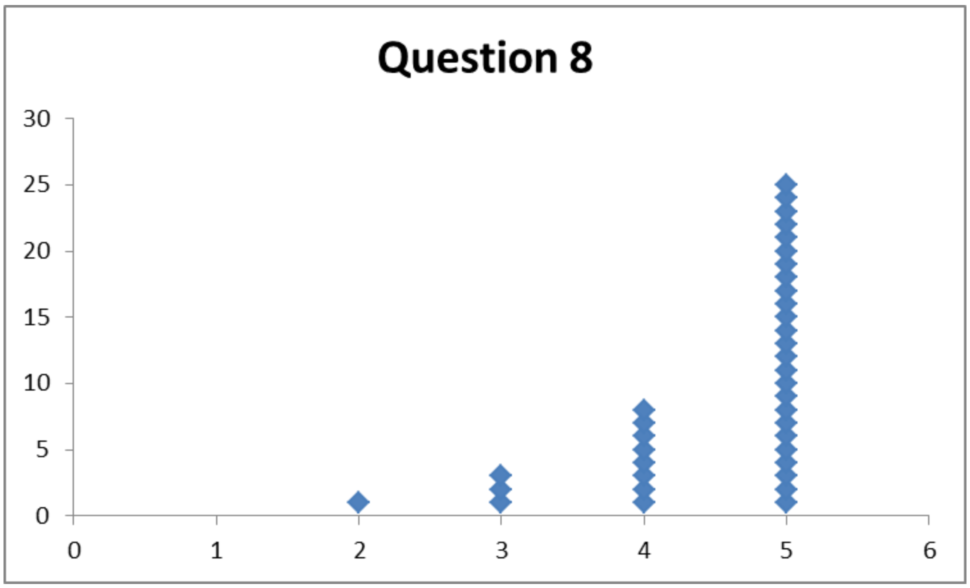
$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 1 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$



8) Given the matrix equation below, identify which variables are free and which variables are leading. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

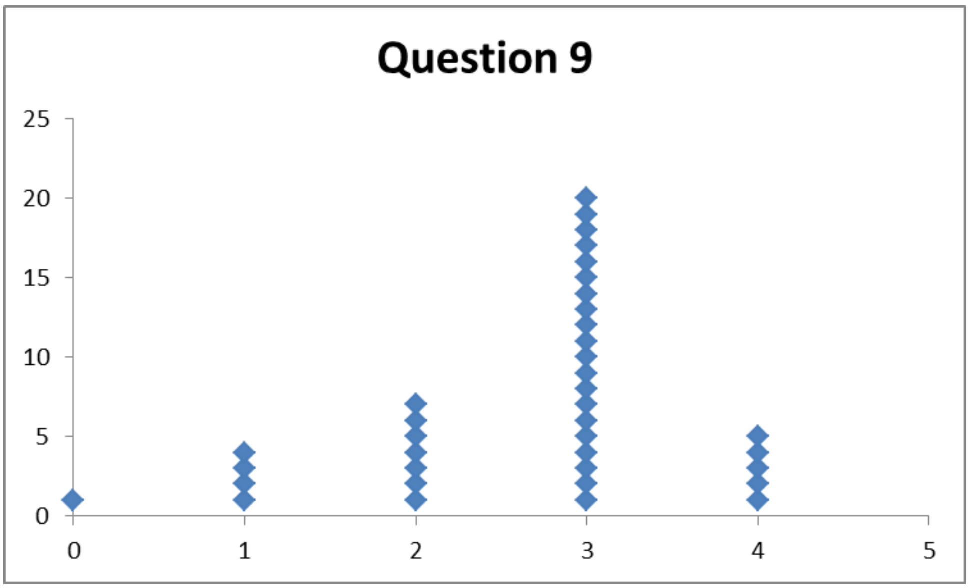
The leading variables are x_1, x_2 and x_5.
The free variables are x_3 and x_4.



9) Which of the following are bases for $\mathbb{R}^2$? Circle them. (4 points)

$$\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\-1\end{bmatrix}\right\} \qquad \left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\} \qquad \left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}3\\4\end{bmatrix},\begin{bmatrix}5\\6\end{bmatrix}\right\} \qquad \left\{\begin{bmatrix}a\\b\end{bmatrix}: a,b\in\mathbb{R}\right\}$$

The second one is not a basis for $\mathbb{R}^2$ because the vectors are in $\mathbb{R}^3$.
The third one is not a basis for anything because they are linearly dependent.
The fourth one is not a basis for anything because they are linearly dependent.



10) Is the following set of vectors linearly dependent or linearly independent? Why? (11 points)

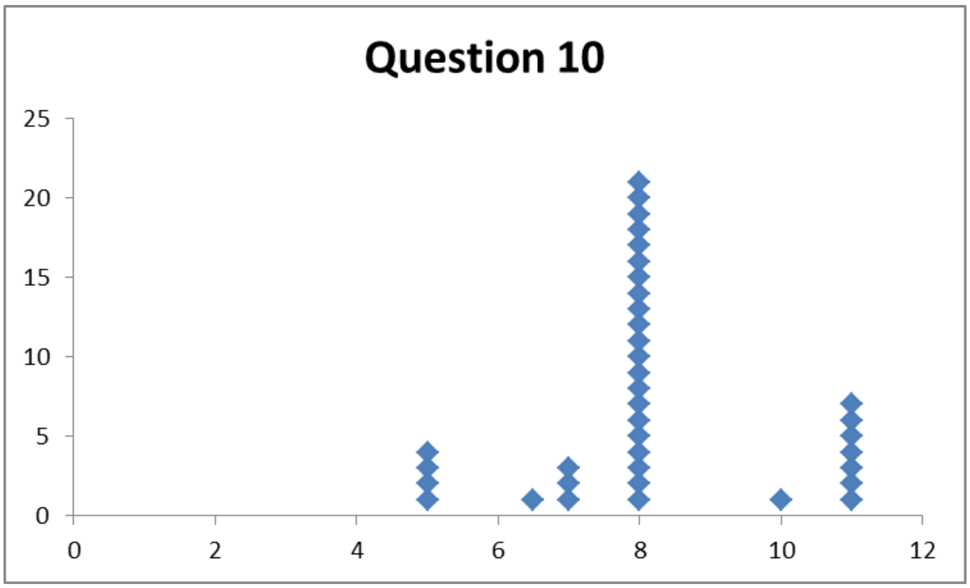
$$\left\{\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\4\\1\end{bmatrix}, \begin{bmatrix}1\\0\\6\end{bmatrix}\right\}$$

These vectors are linearly independent. This can be seen by trying to solve the vector equation below.

$$x_1\begin{bmatrix}2\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\4\\1\end{bmatrix} + x_3\begin{bmatrix}1\\0\\6\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By row reducing the following matrix we see that this equation has only the trivial solution.

$$\begin{bmatrix}2 & 0 & 1\\0 & 4 & 0\\0 & 1 & 6\end{bmatrix} \sim \begin{bmatrix}2 & 0 & 1\\0 & 4 & 0\\0 & 0 & 6\end{bmatrix} \sim \begin{bmatrix}2 & 0 & 0\\0 & 4 & 0\\0 & 0 & 6\end{bmatrix}$$



11) Reduce the following matrix to row reduced echelon form. (20 poin

$$\begin{bmatrix} 2 & 6 & 0 & 4 & 0 \\ 1 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 8 & 0 & 15 & 363 & 20 \end{bmatrix}$$

$R_1 \to \frac{1}{2} R_1$ $\quad R_2 \to R_2 - R_1$ $\quad R_4 \to R_4 - 8R_1$

$$\begin{bmatrix} 2 & 6 & 0 & 4 & 0 \\ 1 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 8 & 0 & 15 & 363 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 1 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 8 & 0 & 15 & 363 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 8 & 0 & 15 & 363 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -24 & 15 & \#\#? & 20 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & -24 & 15 & \#\#? & 20 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 1 & \#\#? & \#\#? & -20/24 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \#\#? & \#\#? & 60/24 \\ 0 & 1 & \#\#? & \#\#? & -20/24 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 60/24 \\ 0 & 1 & 0 & 0 & -20/24 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$R_2 \to \frac{1}{-24} R_2$

$R_1 \to R_1 - 3R_2$

$R_2 \to R_2 - \#\#?\, R_3$
$R_2 \to R_2 - \#\#?\, R_4$
$R_1 \to R_1 - \#\#?\, R_3$
$R_1 \to R_1 - \#\#?\, R_4$