Throughout the test simplify all answers except where stated otherwise. For questions in which the answer is a single number, word, etc, show work or provide an explanation for partial credit; otherwise no credit will be given for incorrect responses.

1) Determine whether or not an inverse exists for the matrix shown below. (5 points)
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
2 & 1 & 0 & 2 \\
3 & 2 & 0 & 3 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]

2) Find the inverse of the matrix shown below. (15 points)
\[
\begin{bmatrix}
1 & 2 & 0 \\
1 & 3 & 0 \\
0 & 7 & 8
\end{bmatrix}
\]
$B_1 = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 8 \end{pmatrix}$.

3) Using the bases above, write the vector below in terms of basis $B_2$. You do not need to simplify your answer. (15 points)

$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

4) A $10 \times 10$ matrix has the following eigenvalues. What can you say about the number of linearly independent eigenvectors? (5 points)

- $\lambda_1 = 2$ with multiplicity 4.
- $\lambda_2 = 1$ with multiplicity 2.
- $\lambda_3 = 0$ with multiplicity 1.
- $\lambda_4 = \sqrt{\pi}$ with multiplicity 1.
- $\lambda_5 = e^{\pi}$ with multiplicity unknown.
5) Find the column space of the matrix below. (5 points)
\[
\begin{bmatrix}
2 & 3 & 5 & 7 \\
1 & 3 & 4 & 9
\end{bmatrix}
\]

6) \(T: \mathbb{R}^{12} \rightarrow \mathbb{R}^{17}\) is a linear operator. It is known that there are at least 6 linearly independent vectors not in the range of \(T\). What can you say about the solution set of the following equation? (15 points)
\[T(\vec{x}) = \vec{0}\]
7) Find all the eigenspaces of the matrix \[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 2 \\
0 & 3 & 2
\end{bmatrix}
\]. (20 points)
8) A $n \times n$ matrix is known to have an eigenvalue of 0. What else can you say? 2 points for each correct statement, -2 points for each incorrect statement. (10 points maximum)

9) $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear operator. It is known that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of $T$, but $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is not an eigenvector of $T$. Illustrate this on the graph below by drawing two vectors: one that could be $T(\begin{bmatrix} 1 \\ 2 \end{bmatrix})$, and one that could be $T(\begin{bmatrix} 4 \\ 2 \end{bmatrix})$. Be sure to label which is which.