Name ____________________________________________________________ Linear Algebra, Test 2

*Throughout the test simplify all answers except where stated otherwise. For questions in which the answer is a single number, word, etc, show work or provide an explanation for partial credit; otherwise no credit will be given for incorrect responses.*

1) Determine whether or not an inverse exists for the matrix shown below. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 3 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

2) Find the inverse of the matrix shown below. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 7 & 8 \end{bmatrix}$$

$$B_1 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix} \right\}, B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix} \right\},$$

3) Using the bases above, write the vector below in terms of basis $B_2$. You do not need to simplify your answer. (15 points)

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{B_1}$$

4) A $10 \times 10$ matrix has matrix has the following eigenvalues. What can you say about the number of linearly independent eigenvectors? (5 points)

$\lambda_1 = 2$ with multiplicity 4.

$\lambda_2 = 1$ with multiplicity 2.

$\lambda_3 = 0$ with multiplicity 1.

$\lambda_4 = \sqrt{\pi}$ with multiplicity 1.

$\lambda_5 = e^{\pi}$ with multiplicity unknown.

5) Find the column space of the matrix below. (5 points)

$$\begin{bmatrix} 2 & 3 & 5 & 7 \\ 1 & 3 & 4 & 9 \end{bmatrix}$$

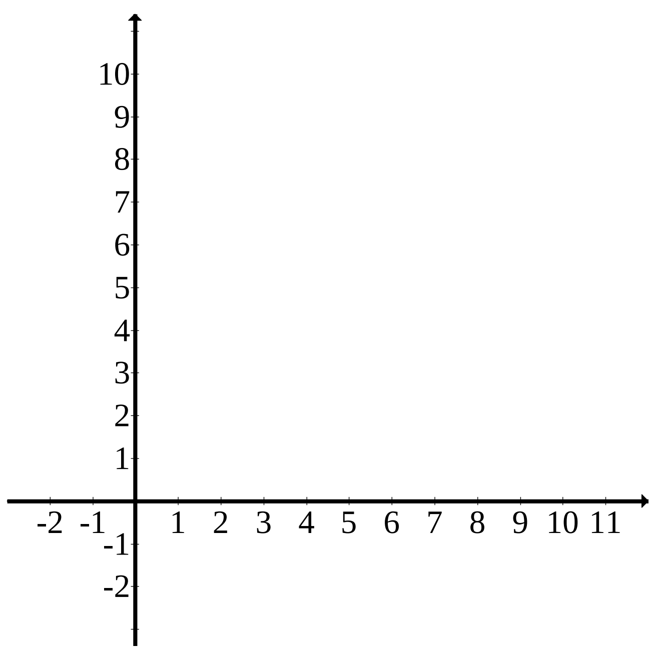
6) $T: \mathbb{R}^{12} \to \mathbb{R}^{17}$ is a linear operator. It is known that there are at least 6 linearly independent vectors not in the range of $T$. What can you say about the solution set of the following equation? (15 points)

$$T(\vec{x}) = \vec{0}$$

7) Find all the eigenspaces of the matrix $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$. (20 points)

8) A $n \times n$ matrix is known to have an eigenvalue of $0$. What else can you say? 2 points for each correct statement, $-2$ points for each incorrect statement. (10 points maximum)

9) $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear operator. It is known that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of $T$, but $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is not an eigenvector of $T$. Illustrate this on the graph below by drawing two vectors: one that could be $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$, and one that could be $T\left(\begin{bmatrix} 4 \\ 2 \end{bmatrix}\right)$. Be sure to label which is which.



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