Name _____Linear Algebra, Test 3, 4/21/2014

*Throughout the test simplify all answers except where stated otherwise. For questions in which the answer is a single number, word, etc, be sure to show your work or provide an explanation.*

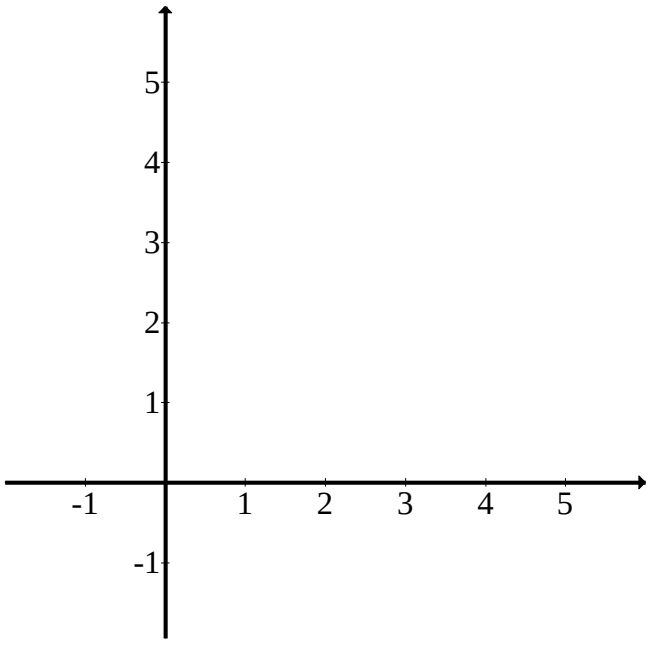
Let $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 3 \\ 4 \\ 1 \end{bmatrix}$

1) Find $\vec{v} \cdot \vec{w}$. (4 points)

2) Find $\|\vec{v}\|$. (5 points)

3) Find the angle between $\vec{v}$ and $\vec{w}$. No need to simplify your answer. (4 points)

4) Consider the $\mathbb{R}^2$ plane below formed by the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. That is, $\text{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}\right)$ is the $x$ axis, while $\text{span}\left(\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right\}\right)$ is the $y$-axis. Project $\vec{v}$ and $\vec{w}$ onto this plane and illustrate the angle between them on the plane. Is this angle the same as your answer to (3) above? (7 points)



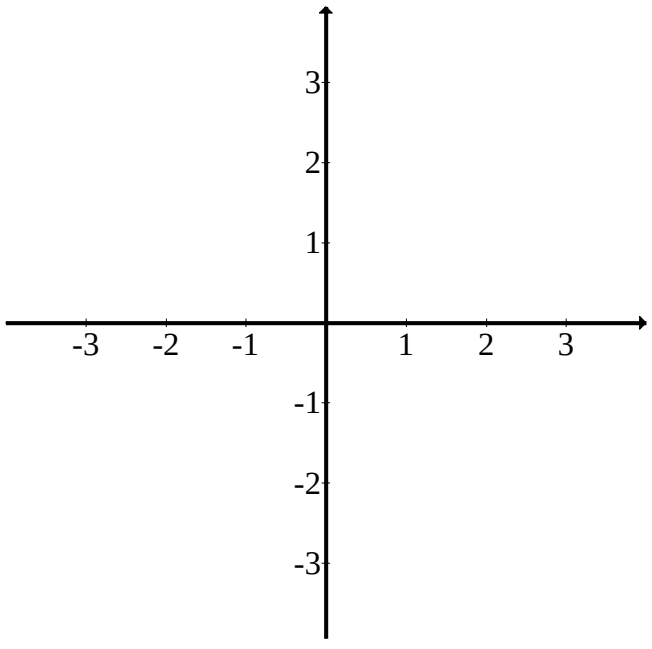
5) Diagonalize the matrix below. (20 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

6) Find an orthogonal basis for the space below. (10 points)

$$\text{span}\left(\left\{\begin{bmatrix}0\\1\\-1\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix},\begin{bmatrix}0\\2\\2\end{bmatrix}\right\}\right)$$

7) Illustrate the orthogonality between (a) the space spanned by $\begin{bmatrix}1\\1\end{bmatrix}$ and (b) a vector of your choice that is orthogonal to (a). (10 points)



8) Find an orthogonal basis for $\mathbb{R}^{77}$. (5 points)

9) Determine whether or not the matrix below is diagonalizable. Justify your answer. (5 points)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

10) Determine whether or not the matrix below exists. Justify your answer. (5 points)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{50}$$

11) Determine whether or not an orthogonal basis exists for the space below. Justify your answer. (5 points)

$$\text{span}\left(\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 11 \\ 2 \\ 3 \end{bmatrix}\right\}\right)$$

12) Find the orthogonal complement of $\text{span}\left(\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}\right)$. (8 points)

13) Express the vector $\begin{bmatrix}1\\2\\3\\-1\end{bmatrix}$ in terms of the basis $\{\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4\} = \left\{\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}\right\}$. (12 points)

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