Name ____ Quiz 3

1) Determine whether or not the vectors below are linearly independent. If they are not linearly independent, find a largest subset of vectors that is linearly independent. Justify your answer.

$$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The fact that each column has a pivot tells us that these vectors are linearly independent.

More detail: Let $A = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Because $A \sim_R B$, we know that $A\vec{x} = \vec{0}$ has the same solution set as $B\vec{x} = \vec{0}$. The only solution to $B\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$, and so the only solution to $A\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$. That is the definition of linear independence.

Some notes on terminology:

"$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ are linearly independent" - OK

"$\left\{\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}\right\}$ is a linearly independent set" - OK

"$\begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}$ is linearly independent" makes NO sense. LI/LD are defined on vectors, not matrices.

"The columns of $\begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}$ are linearly independent" – OK

2) Determine whether or not the vectors below are linearly independent. If they are not linearly independent, find a largest subset of vectors that is linearly independent. Justify your answer.

$$\begin{bmatrix} 4 \\ -2 \\ 5 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \\ -4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 & -1 & 1 \\ -2 & 0 & 2 & 8 \\ 5 & 5 & 0 & 3 \\ -5 & -4 & 1 & 3 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The fact that there is not a pivot in the third column tells us that these vectors are not linearly independent. If we take the columns with pivots, we get a maximum (largest) subset of vectors that is linearly independent:

$$\begin{bmatrix} 4 \\ -2 \\ 5 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 3 \\ 3 \end{bmatrix}$$

More Detail: We take the original vectors, not the reduced columns, because ROW operations change the information provided by the COLUMNS.

Moreover, we see that there is linear dependence between the first three vectors. If we call them $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$, then the reduced form tells us that $\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = \vec{0}$.

Some notes on terminology:

The vector $\begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ itself is linearly independent. What's dependent is the collection of all four together.