

1) Given the basis B , below, for \mathbb{R}^3 and the vector \vec{x} , find a formula for \vec{x} in that basis: $[\vec{x}]_B$

$$B = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

2) Given the bases, B_1 and B_2 , below for \mathbb{R}^3 and the vector \vec{x} , find a formula for \vec{x} in the basis B_2 : $[\vec{x}]_{B_2}$

$$B_1 = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}_{B_1}$$

3) Suppose the matrix A has size 5×13 . That is, 5 rows and 13 columns. It is known that the equation $A\vec{x} = \vec{0}$ has multiple solutions, but that $A\vec{x} = [1 \ 2 \ 3 \ 3 \ 1]^T$ has no solutions. Find the *smallest* possible dimension the null space of A could have.