Name ____ Quiz 5

1) Given the basis B, below, for $\mathbb{R}^3$ and the vector $\vec{x}$, find a formula for $\vec{x}$ in that basis: $[\vec{x}]_B$

$$B = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

$$[I]_B^S = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

2) Given the bases, B_1 and B_2, below for $\mathbb{R}^3$ and the vector $\vec{x}$, find a formula for $\vec{x}$ in the basis B_2: $[\vec{x}]_{B_2}$

$$B_1 = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}_{B_1}$$

$$[I]_{B_1}^S = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix}$$

$$[I]_{B_2}^S = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 7 \\ 2 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 7 \\ 2 & 3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 0 & 2 \\ -1 & 4 & 4 \\ 2 & 1 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

3) Suppose the matrix A has size 5×13. That is, 5 rows and 13 columns. It is known that the equation $A\vec{x} = \vec{0}$ has multiple solutions, but that $A\vec{x} = [1 \quad 2 \quad 3 \quad 3 \quad 1]^T$ has no solutions. Find the *smallest* possible dimension the null space of A could have.

Each free variable in the equation $A\vec{x} = \vec{0}$ corresponds to a dimension in the null space. There are 13 columns, so there are potentially as many as 13 free variables. But there could be as few as $13 - 5 = 8$, if there is a pivot in every row.

However, because there is a vector $\vec{b}$ such that $A\vec{x} = \vec{b}$ has no solutions, we know that at least one row is missing a pivot. Hence there could be as few as $13 - 4 = 9$ free variables. This is the smallest possible dimension of the null space, and is realized with a matrix such as the one below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Half credit: a number that makes sense, but is wrong. (8, 10, 11, 12, 13)
Third credit: a number that isn't remotely possible, but is still a number. (0, 1, 2, 3, 4, 5, 6, 7)
No credit: Wrong type of answer: Something that isn't a whole number.

Note that the information "$A\vec{x} = \vec{0}$ has multiple solutions" is a bit of a red herring. Because $13 > 5$, it should be obvious that $A\vec{x} = \vec{0}$ has multiple solutions.