

1) Find the determinant of the matrix below.

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 2 \\ 4 & -2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 3 & -1 \\ 2 & 2 \end{vmatrix} = 2 + 4 + 4(6 + 2) = 38$$

2) Find the determinant of the matrix below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 5 & 5 & 5 & 5 & 5 \\ 4 & 4 & 4 & 4 & 0 \\ 3 & 3 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 5 & 5 & 5 & 5 & 5 \\ 4 & 4 & 4 & 4 & 0 \\ 3 & 3 & 3 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 \\ 5 & 5 & 5 & 5 & 5 \end{vmatrix} = -1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = -120$$

3) Suppose the matrix A has size 6×6 . That is, 6 rows and 6 columns. It is known that the equation $A\vec{x} = \vec{b}$ has a solution when $b = [1 \ 2 \ 5 \ 4 \ 5 \ 1]^T$. It is also known that $|A| = 3$. How many solutions are there to $A\vec{x} = \vec{b}$?

The fact that $|A| = 3$ tells us that A is invertible, meaning that the solution to $A\vec{x} = \vec{b}$ is unique. That is:

There is one solution to $A\vec{x} = \vec{b}$.