Name ____ Quiz 7 Solutions

$V = \mathbb{R}^3$ with basis $B_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$

$W = \mathbb{R}^2$ with basis $B_2 = \left\{ \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix} \right\}$

T is a linear transformation from V to W, and is given by:

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}_S \right) = \begin{bmatrix} x + y \\ z \end{bmatrix}_S$$

1) Using the information above, find a formula for T that allows us to compute $T(\vec{x})$ when $\vec{x}$ is expressed in the natural basis of V. You do not need to simplify the formula.

$$\begin{bmatrix} 5 & 4 \\ 3 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2) Suppose a linear transformation T goes from $\mathbb{R}^{12}$ to $\mathbb{R}^4$, and it is known that $A\vec{x} = \vec{b}$ has no solutions when $\vec{b} = [1 \quad 2 \quad 3 \quad 5]^T$. What is the determinant of $[T]^{\mathrm{T}}[T]$?

(Here A is the matrix representing T)

Because $A\vec{x} = \vec{b}$ has no solutions, we know that there is a row without a pivot. The matrix A has 4 rows and 12 columns. So A has rank at most 3, which means that $A^T A$ has rank at most 3. That means that $|A^T A| = 0$.