Name ____________________________________________________________ Test 1, Fall 2017

Use the system of equations below for the following FOUR problems.

$$-5x_1 - 3x_2 = 4$$
$$2x_2 = 10$$

1) Solve the system of equations.

(4 points)

2) Write the system of equations a matrix equation $A\vec{x} = \vec{b}$.

(4 points)

3) Write the system of equations as a vector equation $\vec{v}_1 c_1 + \vec{v}_2 c_2 = \vec{b}$.

(4 points)

4) Let $A$ be the $2 \times 2$ matrix with complex entries given by $A = \begin{bmatrix} 0 & a+bi \\ 0 & 0 \end{bmatrix}$. Show that $(\bar{A})^T = \overline{A^T}$.

(4 points)

5) Answer the following as true or false. A statement is true if it is *always* true; false if it is *ever* false.

(Assume sizes are such that addition and multiplication operations make sense) (2 points each)

T F a) Suppose $A$ is a square matrix with two equal columns. Then $A$ is invertible.

T F b) Suppose $A$ and $B$ are square matrices. Then $AB = BA$.

T F c) Suppose $A$ and $B$ are square matrices. Then $A^2 - B^2 = (A - B)(A + B)$.

T F d) Suppose $A$ is not a square matrix. Then $A$ is symmetric.

T F e) Suppose $A$ is not a square matrix. Then $A^T A$ is symmetric.

T F f) Suppose $A$ and $C$ are square matrices, but $B$ is not. Then $(ABC)^T = C^T B^T A^T$.

T F g) Suppose $A$, $B$, and $C$ are invertible matrices. Then $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

6) Find $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 5 & 4 \end{bmatrix}^T$

(4 points)

7) Find an example of a $2 \times 4$ matrix $A$ such that $AA^T$ is nonsingular.

(6 points)

8) Find a formula for the quadratic form $q = q(x, y, z)$ with the matrix representation $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

(4 points)

9) Given the matrix $A$ below, does the nonhomogeneous system of equations $A\vec{x} = \vec{b}$ have a solution for every choice of $b$? Justify your answer. (6 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

10) Partition the matrix below so that it has 6 parts.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 2 \\ 3 & 6 & 9 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(4 points)

11) Let $A, B, C$, and $X$ be matrices of appropriate sizes. Assume everything is invertible. Solve the equation $B(X + A)^{-1} = C$ for $X$. Show your work.
(6 points)

12) Determine if $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ are orthogonal. Show your work.
(4 points)

13) Give an example of an infinite set of vectors in $\mathbb{R}^3$ that do not form a vector space.
(4 points)

14) Give an example of a vector space that is not $\mathbb{R}^n$ for any $n$.
(4 points)

15) Find the null space of $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 4 \end{bmatrix}$

(10 points)

16) Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 4 \\ 0 & 2 & 5 \end{bmatrix}$

(10 points)

17) You know that the matrix equation $A\vec{x} = \vec{0}$ has more than one solution. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 8 points maximum)

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