Use the system of equations below for the following FOUR problems.

\[-5x_1 - 3x_2 = 4\]
\[2x_2 = 10\]

1) Solve the system of equations.
(4 points)

2) Write the system of equations a matrix equation \(A\vec{x} = \vec{b}\).
(4 points)

3) Write the system of equations as a vector equation \(\vec{v}_1c_1 + \vec{v}_2c_2 = \vec{b}\).
(4 points)

4) Let \(A\) be the 2 × 2 matrix with complex entries given by \(A = \begin{bmatrix} 0 & a + bi \\ 0 & 0 \end{bmatrix}\). Show that \((A^\dagger)^T = \overline{A^T}\).
(4 points)
5) Answer the following as true or false. A statement is true if it is always true; false if it is ever false. (Assume sizes are such that addition and multiplication operations make sense) (2 points each)

T F  a) Suppose $A$ is a square matrix with two equal columns. Then $A$ is invertible.
T F  b) Suppose $A$ and $B$ are square matrices. Then $AB = BA$.
T F  c) Suppose $A$ and $B$ are square matrices. Then $A^2 - B^2 = (A - B)(A + B)$.
T F  d) Suppose $A$ is not a square matrix. Then $A$ is symmetric.
T F  e) Suppose $A$ is not a square matrix. Then $A^T A$ is symmetric.
T F  f) Suppose $A$ and $C$ are square matrices, but $B$ is not. Then $(ABC)^T = C^T B^T A^T$.
T F  g) Suppose $A$, $B$, and $C$ are invertible matrices. Then $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$.

6) Find $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 5 & 4 \end{bmatrix}^T$ (4 points)

7) Find an example of a $2 \times 4$ matrix $A$ such that $AA^T$ is nonsingular. (6 points)
8) Find a formula for the quadratic form \( q = q(x, y, z) \) with the matrix representation \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & -2
\end{bmatrix}.
\]

(4 points)

9) Given the matrix \( A \) below, does the nonhomogeneous system of equations \( A\vec{x} = \vec{b} \) have a solution for every choice of \( \vec{b} \)? Justify your answer. (6 points)

\[
A = \begin{bmatrix}
1 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 1 & 2 \\
0 & 0 & 0 & 2 & 4
\end{bmatrix}
\]

10) Partition the matrix below so that it has 6 parts.

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 2 & 2 & 2 \\
3 & 6 & 9 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(4 points)
11) Let $A, B, C$, and $X$ be matrices of appropriate sizes. Assume everything is invertible. Solve the equation $B(X + A)^{-1} = C$ for $X$. Show your work.

(6 points)

12) Determine if $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ are orthogonal. Show your work.

(4 points)

13) Give an example of an infinite set of vectors in $\mathbb{R}^3$ that do not form a vector space.

(4 points)

14) Give an example of a vector space that is not $\mathbb{R}^n$ for any $n$.

(4 points)
15) Find the null space of \[
\begin{bmatrix}
1 & 2 & -2 \\
0 & 1 & 4 \\
\end{bmatrix}
\]
(10 points)
16) Find the inverse of \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 4 \\
0 & 2 & 5 \\
\end{bmatrix}
\]

(10 points)
17) You know that the matrix equation $A\vec{x} = \vec{0}$ has more than one solution. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 8 points maximum)