Name

Non-calculator portion. Turn this portion in before you take out your technology.

1) An equation $A\vec{x} = \vec{b}$ has the solution given below, where s_1 and s_2 are free variables. Express the solution as a meaningful linear combination of vectors.

$$x_1 = -1 + 2s_1 x_2 = 3s_1 + 2s_2 x_3 = s_1 - s_2$$

(6 points)

2) Expand the set below to give a basis for \mathbb{R}^2 .

 $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$

(4 points)

3) A 8 \times 5 matrix has a null space of dimension 3. What is the rank of *A*? (6 points)

4) Suppose the matrix A has size 11×7 . That is, 11 rows and 7 columns. It is known that the equation $A\vec{x} = \vec{0}$ has multiple solutions, but that $A\vec{x} = \begin{bmatrix} 1 & 2 & 3 & 3 & 1 \end{bmatrix}^T$ has no solutions. Find the *smallest* possible dimension the null space of A could have. (6 points) 5) Find the determinant of the matrix below.

$$\begin{bmatrix} -4 & 2 & -2 \\ 1 & 0 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

(10 points)

6) Find the matrix A so that the linear transformation below is given by $T(\vec{x}) = A\vec{x}$. $T: \mathbb{R}^3 \to \mathbb{R}^2$

$$\begin{array}{c} T \colon \mathbb{R}^3 \to \mathbb{R}^2 \\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_2 - 5x_3 \\ 7x_3 \end{bmatrix} \end{aligned}$$

(6 points)

7) Let B be defined below. Find the change of basis matrix $[I]_B^S$.

$$B = \left\{ \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \begin{bmatrix} 4\\4\\6 \end{bmatrix}, \begin{bmatrix} 0\\2\\5 \end{bmatrix} \right\}$$

(4 points)

8) Answer the following as true or false. A statement is true if it is *always* true; false if it is *ever* false. (2 points each)

- T F a) Every matrix A has a determinant.
- T F b) The range, T(V), of a linear transformation $T: V \to W$ is a vector space.
- T F c) If a linear transformation $T: V \rightarrow W$ is one-to-one, then it is onto.
- T F d) If $A\vec{x} = \vec{b}$ can be solved for every choice of \vec{b} , then the linear transformation given by $T(\vec{x}) = A\vec{x}$ is onto.
- T F e) Using matrices, we can write down a formula for the solution \vec{x} to $A\vec{x} = \vec{b}$ no matter what \vec{b} is.

9) Let T be the linear transformation below. Prove that T is onto by either using some theorem/concept or by finding the input \vec{x} that maps to the output $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

$$T: \mathbb{R}^3 \to \mathbb{R}^2$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$$

(4 points)

10) Let A be an $n \times n$ matrix. You know that the matrix equation $A\vec{x} = \vec{0}$ has more than one solution. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 12 points maximum)

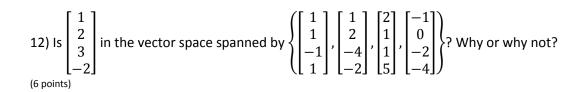
Name ____

Technology portion: After you turn in the non-calculator portion, you may take out your technology and finish this portion. If you're using a phone or tablet, leave it flat on the desk.

Use the matrix A, below, for the problems on this page.

A =	[1	1	2	-1	1]
Λ —	1	2	1	0	2
А —	-1	-4	1	-2	3
	[1	-2	5	-4	-2

11) Find the null space of *A*. (6 points)



13) Find a subset of the vectors below that is a basis of \mathbb{R}^3

([1]		[3]		[11]		[7]		[3])	۱
ł	2	,	2	,	[11 10 7	,	6	,	2		ł
(2		1		7		4		2	J)

(6 points)

14) Let B_1 and B_2 be defined below. Find the change of basis matrix $[I]_{B_1}^{B_2}$.

$$B_{1} = \left\{ \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \begin{bmatrix} 4\\4\\6 \end{bmatrix}, \begin{bmatrix} 0\\2\\5 \end{bmatrix} \right\}, \qquad B_{2} = \left\{ \begin{bmatrix} 1\\0\\3 \end{bmatrix}, \begin{bmatrix} 4\\-3\\2 \end{bmatrix}, \begin{bmatrix} 1\\1\\1 \end{bmatrix} \right\}$$

(8 points)

15) Define the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ via $T(\vec{x}) = A\vec{x}$. What is $T(\begin{bmatrix} 1 & 2 & 4 & 2 \end{bmatrix}^T)$? $A = \begin{bmatrix} 6 & 1 & 2 & 1 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 \end{bmatrix}$

(6 points)

16) Define the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^5$ via $T(\vec{x}) = A\vec{x}$ where A is defined below. Is T one-toone? Why or why not?

1 3 0 0	2 2 1 3	3 1 2 6
0	3	6
l_1°	1	1

(6 points)