Name ____ Test 2, Fall 2017

Non-calculator portion. Turn this portion in before you take out your technology.

1) An equation $A\vec{x} = \vec{b}$ has the solution given below, where s_1 and s_2 are free variables. Express the solution as a meaningful linear combination of vectors.

$$x_1 = -1 + 2s_1$$
$$x_2 = 3s_1 + 2s_2$$
$$x_3 = s_1 - s_2$$

(6 points)

2) Expand the set below to give a basis for $\mathbb{R}^2$.

$$\left\{\begin{bmatrix} 1 \\ -3 \end{bmatrix}\right\}$$

(4 points)

3) A 8×5 matrix has a null space of dimension 3. What is the rank of A?

(6 points)

4) Suppose the matrix A has size 11×7. That is, 11 rows and 7 columns. It is known that the equation $A\vec{x} = \vec{0}$ has multiple solutions, but that $A\vec{x} = [1 \quad 2 \quad 3 \quad 3 \quad 1]^T$ has no solutions. Find the *smallest* possible dimension the null space of A could have.

(6 points)

5) Find the determinant of the matrix below.

$$\begin{bmatrix} -4 & 2 & -2 \\ 1 & 0 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

(10 points)

6) Find the matrix A so that the linear transformation below is given by $T(\vec{x}) = A\vec{x}$.

$$T: \mathbb{R}^3 \to \mathbb{R}^2$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_2 - 5x_3 \\ 7x_3 \end{bmatrix}$$

(6 points)

7) Let B be defined below. Find the change of basis matrix $[I]_B^S$.

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix} \right\}$$

(4 points)

8) Answer the following as true or false. A statement is true if it is *always* true; false if it is *ever* false.

(2 points each)

T F a) Every matrix A has a determinant.

T F b) The range, $T(V)$, of a linear transformation $T: V \to W$ is a vector space.

T F c) If a linear transformation $T: V \to W$ is one-to-one, then it is onto.

T F d) If $A\vec{x} = \vec{b}$ can be solved for every choice of $\vec{b}$, then the linear transformation given by $T(\vec{x}) = A\vec{x}$ is onto.

T F e) Using matrices, we can write down a formula for the solution $\vec{x}$ to $A\vec{x} = \vec{b}$ no matter what $\vec{b}$ is.

9) Let T be the linear transformation below. Prove that T is onto by either using some theorem/concept or by finding the input $\vec{x}$ that maps to the output $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

$$T: \mathbb{R}^3 \to \mathbb{R}^2$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_1 \\ 2x_2 \end{bmatrix}$$

(4 points)

10) Let A be an $n \times n$ matrix. You know that the matrix equation $A\vec{x} = \vec{0}$ has more than one solution. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 12 points maximum)

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Technology portion: After you turn in the non-calculator portion, you may take out your technology and finish this portion. If you're using a phone or tablet, leave it flat on the desk.

Use the matrix A, below, for the problems on this page.

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 & 1 \\ 1 & 2 & 1 & 0 & 2 \\ -1 & -4 & 1 & -2 & 3 \\ 1 & -2 & 5 & -4 & -2 \end{bmatrix}$$

11) Find the null space of A.

(6 points)

12) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \end{bmatrix}$ in the vector space spanned by $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \\ -4 \end{bmatrix} \right\}$? Why or why not?

(6 points)

13) Find a subset of the vectors below that is a basis of $\mathbb{R}^3$

$$\left\{\begin{bmatrix}1\\2\\2\end{bmatrix},\begin{bmatrix}3\\2\\1\end{bmatrix},\begin{bmatrix}11\\10\\7\end{bmatrix},\begin{bmatrix}7\\6\\4\end{bmatrix},\begin{bmatrix}3\\2\\2\end{bmatrix}\right\}$$

(6 points)

14) Let B_1 and B_2 be defined below. Find the change of basis matrix $[I]_{B_1}^{B_2}$.

$$B_1 = \left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}4\\4\\6\end{bmatrix},\begin{bmatrix}0\\2\\5\end{bmatrix}\right\}, \qquad B_2 = \left\{\begin{bmatrix}1\\0\\3\end{bmatrix},\begin{bmatrix}4\\-3\\2\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$$

(8 points)

15) Define the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ via $T(\vec{x}) = A\vec{x}$. What is $T([1 \quad 2 \quad 4 \quad 2]^T)$?

$$A = \begin{bmatrix} 6 & 1 & 2 & 1 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 \end{bmatrix}$$

(6 points)

16) Define the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^5$ via $T(\vec{x}) = A\vec{x}$ where A is defined below. Is T one-to-one? Why or why not?

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

(6 points)