\Name ____ Test 3, Fall 2017

The entire test is non-calculator. If a question says to "find a formula for…" you do not need to simplify anything or do any arithmetic. If it can be plugged into Wolfram Alpha using commands we've used in class, your answer is good enough!

Choose EIGHT of the first 10 problems to complete for 10 points each. Problem 11 cannot be skipped. Cross out the two problems you're skipping. Failure to do so will result in a -8 point penalty per question.

1) Let T be defined as below. Find a formula for $T^{-1}\left(\begin{bmatrix}7\\8\\9\end{bmatrix}\right)$.

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \mapsto \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\0 & 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

2) The linear transformations T is defined as below. Note that V has basis $\left\{\begin{bmatrix}2\\0\\1\end{bmatrix}, \begin{bmatrix}1\\3\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\2\end{bmatrix}\right\}$.

Find a formula for $\left[T\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right)\right]_S$

$$T\colon \mathbb{R}^3 \to V$$
$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \mapsto \begin{bmatrix}1 & 4 & 3\\4 & 5 & -2\\3 & 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

3) The linear transformation T is defined as below. Note that V has basis $\left\{\begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$ and W has basis $\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$. Draw the diagram relating $V, W, \mathbb{R}^2$, and $\mathbb{R}^3$ then find a formula for $[T]_S^S$.

$$T: V \to W$$
$$\begin{bmatrix}x_1\\x_2\end{bmatrix} \mapsto \begin{bmatrix}1 & 5\\2 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

4) The linear transformations T and S are defined as below. Note that V has basis $\left\{\begin{bmatrix}2\\0\\1\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}-1\\1\\1\end{bmatrix}\right\}$.

Find a formula for $[S \circ T]_S^S$.

$$T: \mathbb{R}^3 \to \mathbb{R}^2$$
$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \mapsto \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$S: \mathbb{R}^2 \to V$$
$$\begin{bmatrix}x_1\\x_2\end{bmatrix} \mapsto \begin{bmatrix}1 & 2\\2 & 5\\2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

5) Given the matrix below, find the smaller of the two eigenspaces.

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 3 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

6) Apply the Gram-Schmidt Process to find an orthonormal basis for S.

$$S = \text{span}\left(\left\{\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}3\\4\\3\end{bmatrix}, \begin{bmatrix}4\\5\\10\end{bmatrix}\right\}\right)$$

7) Diagonalize $\begin{bmatrix} 7 & -10 \\ 2 & -2 \end{bmatrix}$. Make sure your answer is in the form "$A = PDP^{-1}$".

8) Suppose $T: \mathbb{R}^5 \to \mathbb{R}^7$ is one-to-one. How many solutions does $[T]\vec{x} = \vec{b}$ have? Justify your answer.

9) Suppose a 5×5 matrix A has determinant 2. How many solutions does the system below have?
$[A|\vec{v}]\vec{x} = \vec{b}$. Here $\vec{v} = [1 \quad 2 \quad 3 \quad 4 \quad 5]^T$ and $\vec{b} = [1 \quad 2 \quad 3 \quad 4 \quad 6]^T$

10) Find the eigenvalue λ so that $\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 3 & 0 & 0 \\ -1 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, then write down a formula for a and b.

11) Let A be an $n \times n$ matrix. You know that $\lambda = 0$ is an eigenvalue. What else can you say?

(4 points per insightful statement; 1 point per obvious statement; every incorrect statement nullifies a correct statement. 20 points maximum)