Name ____ Quiz 3

For these problems define $\vec{v}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, \vec{v}_2 = \begin{bmatrix}1\\0\\1\end{bmatrix}, \vec{v}_3 = \begin{bmatrix}0\\1\\2\end{bmatrix}$, $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, and $[B] = [\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3]$. Note that B is a basis for the vector space $\mathbb{R}^3$ and $[B]$ is a 3×3 matrix who's columns are the vectors $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

1) Find the linear combination $2\vec{v}_1 - 3\vec{v}_2 + \vec{v}_3$

$$2\vec{v}_1 - 3\vec{v}_2 + \vec{v}_3 = 2\begin{bmatrix}1\\2\\3\end{bmatrix} - 3\begin{bmatrix}1\\0\\1\end{bmatrix} + \begin{bmatrix}0\\1\\2\end{bmatrix} = \begin{bmatrix}2\\4\\6\end{bmatrix} + \begin{bmatrix}-3\\0\\-3\end{bmatrix} + \begin{bmatrix}0\\1\\2\end{bmatrix} = \begin{bmatrix}-1\\5\\5\end{bmatrix}$$

2) Given the vector $\vec{x}_B = \begin{bmatrix}2\\-3\\1\end{bmatrix}_B$, find $\vec{x}_S$.

$$\vec{x}_x = \begin{bmatrix}1 & 1 & 0\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\-3\\1\end{bmatrix}_B = \begin{bmatrix}-1\\5\\5\end{bmatrix}_S$$

That looks surprisingly similar to the answer to #1. Do you know why?

3) If we were to row reduce $[B]$, how many rows would have a pivot?

All 3 rows would have pivots.

Please note that you can solve this problem conceptually. You will not have time on a test to actually row reduce $[B]$.

4) What is $\dim\big(RS([B])\big)$?

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5) How many solutions does $[B]\vec{x} = \vec{0}$ have?

One unique solution.