Name _____ Quiz 4

For these problems define $B_1 = \left\{\begin{bmatrix}1\\5\end{bmatrix}, \begin{bmatrix}2\\6\end{bmatrix}\right\}$ and $B_2 = \left\{\begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}\right\}$. A linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}_S\right) = \begin{bmatrix}x_1 + x_2\\x_2\end{bmatrix}_S$

1) Find an expression that gives $[T]_S^S$

$$[T]_S^S = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

2) Find an expression that gives $[T]_{B_1}^{B_2}$

$$[T]_{B_1}^{B_2} = \begin{bmatrix}0 & 1\\2 & 3\end{bmatrix}^{-1}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix}$$

3) A 5×7 matrix has a null space of dimension 3. When it is row reduced, how many rows of zeroes will there be?

One

A null space of dimension 3 means there are 3 free variables. From 7 columns total, this means there are 4 pivots. That means of the 5 rows, there is one left with a row of zeroes.