Name ______________________________________________________________________ Quiz 5

Find all the eigenvalues of the matrix below, then choose one eigenvalue and find its corresponding eigenspace.

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 5 & 3 & -3 \end{bmatrix}$$

$$|xI - A| = \begin{vmatrix} x-2 & 0 & 0 \\ 1 & x-1 & 0 \\ -5 & -3 & x+3 \end{vmatrix} = (x-2)(x-1)(x+3)$$

$\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = -3$

For $\lambda_1 = 2$ we have:

$$\begin{bmatrix} 2-2 & 0 & 0 \\ 1 & 2-1 & 0 \\ -5 & -3 & 2+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -5 & -3 & 5 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

The elements of the null then look like $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{5}{2}x_3 \\ -\frac{5}{2}x_3 \\ x_3 \end{bmatrix}$ If we choose $x_3 = 2$ we get a good basis vector for the Eigenspace:

$$span\left(\left\{\begin{bmatrix} 5 \\ -5 \\ 2 \end{bmatrix}\right\}\right)$$

For $\lambda_2 = 1$ we have:

$$\begin{bmatrix} 1-2 & 0 & 0 \\ 1 & 1-1 & 0 \\ -5 & -3 & 1+3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ -5 & -3 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

The elements of the null then look like $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{4}{3}x_3 \\ x_3 \end{bmatrix}$ If we choose $x_3 = 3$ we get a good basis vector for the Eigenspace:

$$span\left(\left\{\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}\right\}\right)$$

For $\lambda_2 = -3$ we have:

$$\begin{bmatrix} -3-2 & 0 & 0 \\ 1 & -3-1 & 0 \\ -5 & -3 & -3+3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 1 & -4 & 0 \\ -5 & -3 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The elements of the null then look like $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix}$ If we choose $x_3 = 1$ we get a good basis vector for the Eigenspace:

$$span\left(\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}\right)$$