

Turn $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$ into an orthogonal basis of \mathbb{R}^3 .

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}; \text{proj}_{\vec{b}_1}(\vec{u}_2) = \frac{1+2}{1+1} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \\ 0 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 3/2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix}, \text{ but if it doesn't need to be normalized, let's scale it by 2: } \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}; \text{proj}_{\vec{b}_1}(\vec{u}_3) = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}; \text{proj}_{\vec{b}_2}(\vec{u}_3) = \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

$$\vec{b}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

The orthogonal basis is then:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$$