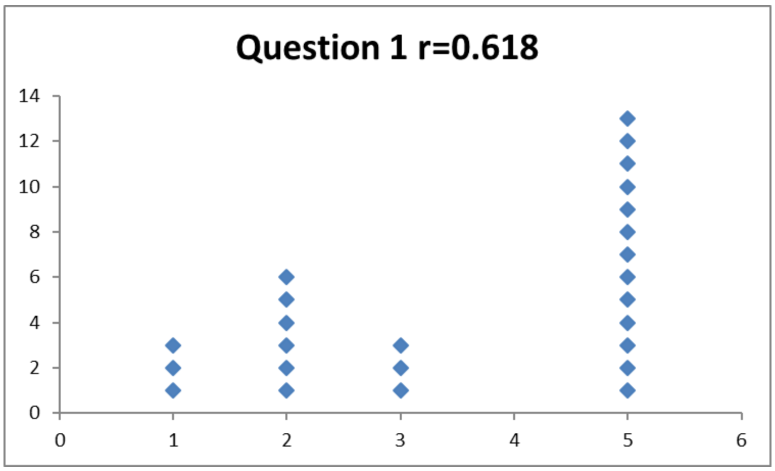
Name ____ Test 1

1) Given the system of equations below, find the corresponding matrix equation. (5 points)

$$7x - y = 1$$
$$y = 5$$

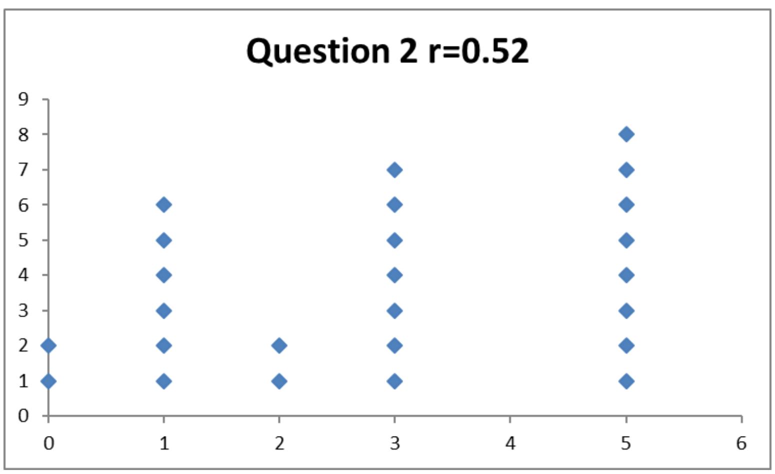
$$\begin{bmatrix} 7 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$



2) In the equation below, circle all answers that describe how A relates to B. (5 points)

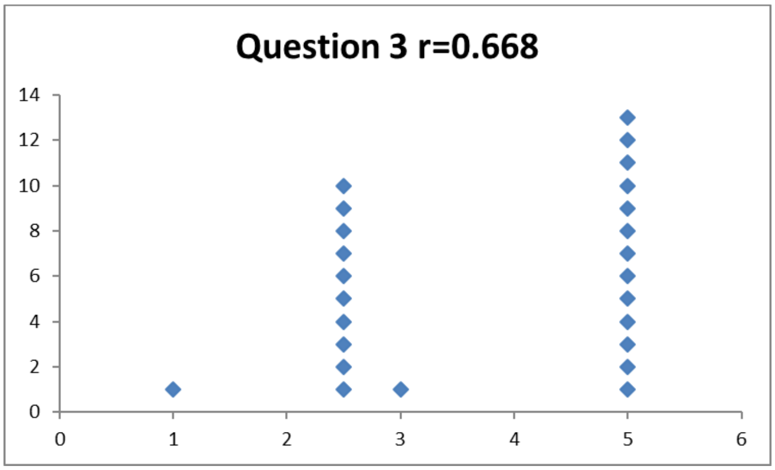
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} A = B$$

(A) Matrix B is the same as matrix A with rows 2 and 4 swapped.
(B) Matrix B is the same as matrix A with rows 1 and 3 swapped.
(C) Matrix B is the same as matrix A with columns 2 and 4 swapped.
(D) Matrix B is the same as matrix A with columns 1 and 3 swapped.
(E) Matrix B is the same as matrix A with row 2 multiplied by 2
(F) Matrix B is the same as matrix A with row 2 multiplied by ½
(G) Matrix B is the same as matrix A with row 4 multiplied by 2
(H) Matrix B is the same as matrix A with row 4 multiplied by ½
(I) Matrix B is the same as matrix A with column 2 multiplied by 2
(J) Matrix B is the same as matrix A with column 2 multiplied by ½
(K) Matrix B is the same as matrix A with column 4 multiplied by 2
(L) Matrix B is the same as matrix A with column 4 multiplied by ½



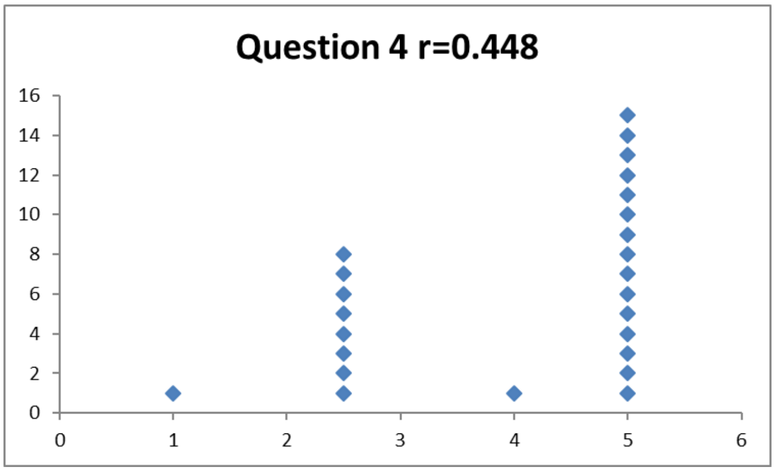
3) Given $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$, how many solutions does $A\vec{x} = \vec{0}$ have? (5 points)

Just one, $\vec{x} = \vec{0}$. (There are no free variables)



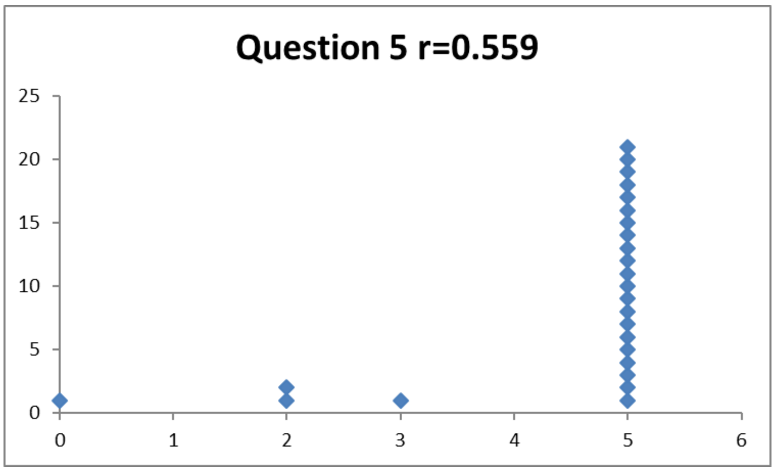
4) Given $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$, how many solutions does $A\vec{x} = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 3 \end{bmatrix}$ have? (5 points)

No solutions. (The last "equation" is $0 = 3$)



5) Find the length of $\begin{bmatrix}1\\0\\2\\5\end{bmatrix}$. (5 points)

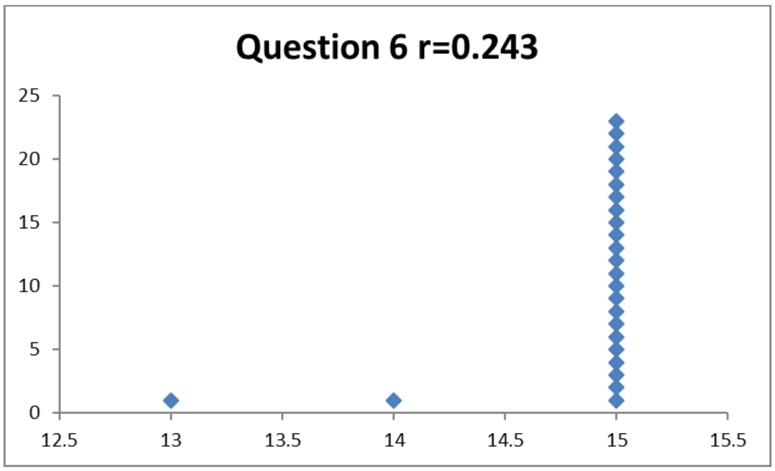
$$\sqrt{1^2+2^2+5^2}=\sqrt{1+4+25}=\sqrt{30}$$



6) Multiply the two matrices as indicated below. (15 points)

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 7 \\ 2 & 2 & 3 \\ 4 & 2 & 3 \end{bmatrix}$$

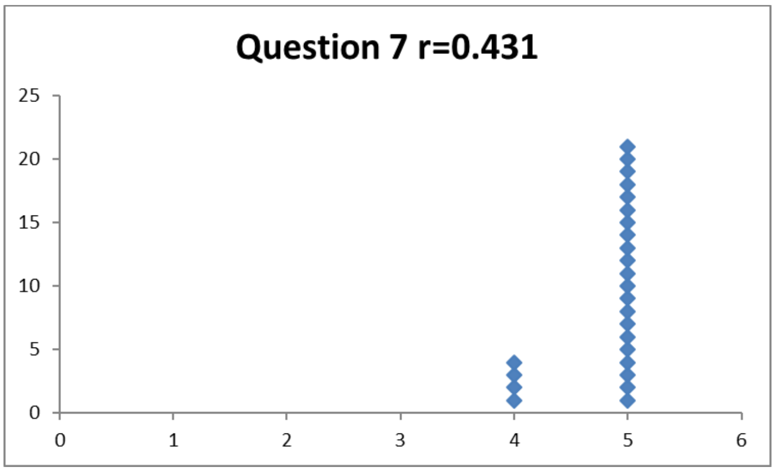
$$\begin{bmatrix} 13 & 8 & 16 \\ 10 & 10 & 23 \end{bmatrix}$$



7) Add the two matrices as indicated below. (5 points)

$$\begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

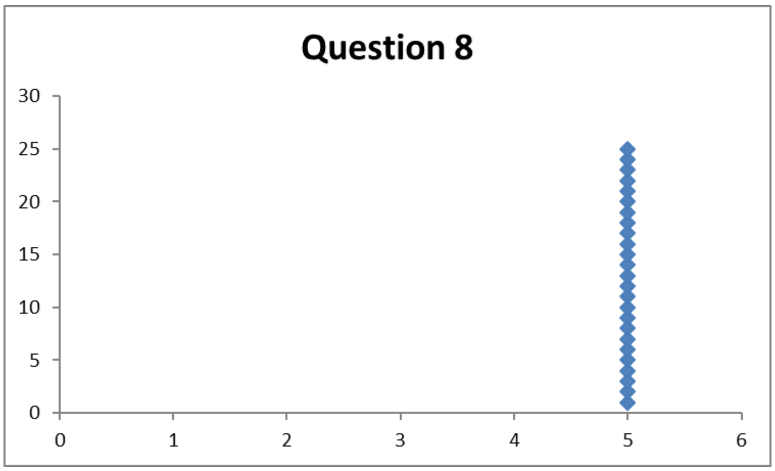
$$\begin{bmatrix} 3 & 3 \\ 4 & 5 \end{bmatrix}$$



8) Find the transpose of the matrix as indicated below. (5 points)

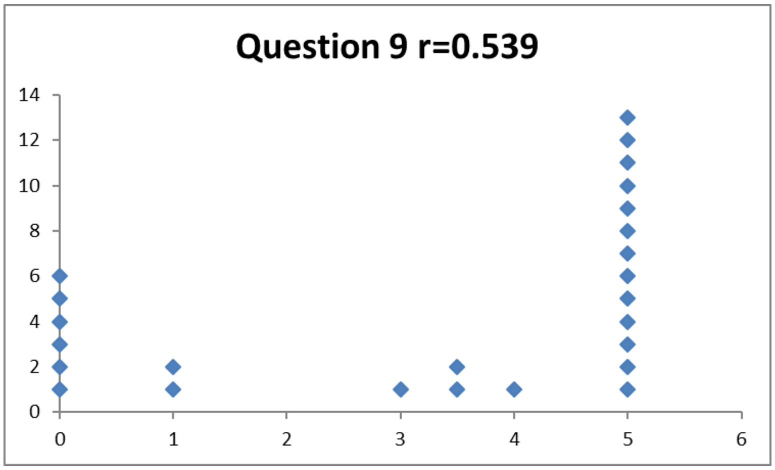
$$\begin{bmatrix} 1 & 2 \\ 8 & -2 \end{bmatrix}^T$$

$$\begin{bmatrix} 1 & 8 \\ 2 & -2 \end{bmatrix}$$



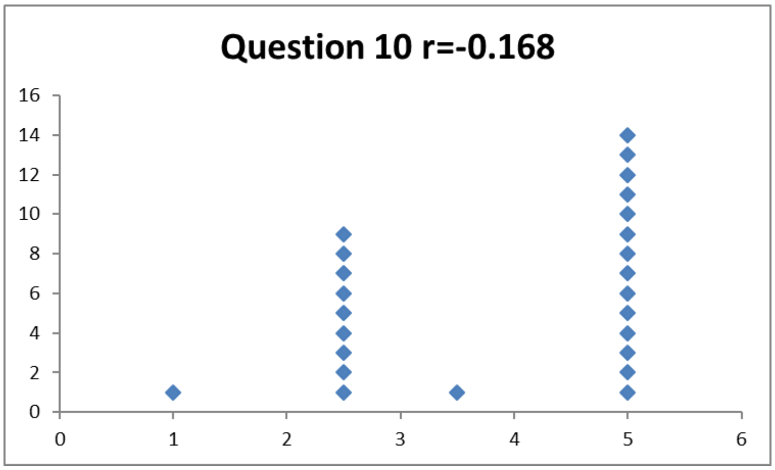
9) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$, find the quadratic form that comes from this matrix. (5 points)

$$f(x, y) = 2x^2 + 6xy + 6y^2$$



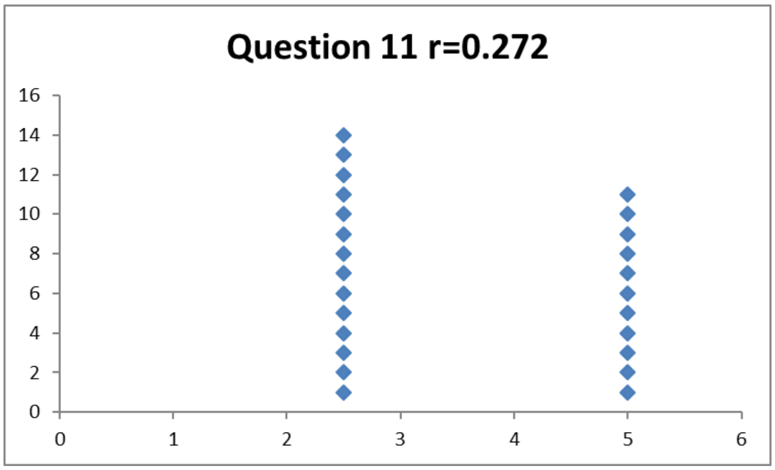
10) Let A be a 2×2 singular matrix. How many solutions does $A\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ have? (5 points)

Infinitely many (There is a free variable)



11) Assume A is a 5×5 matrix. If A is not a product of elementary matrices, how many solutions does the matrix equation $A\vec{x} = \vec{0}$ have? (5 points)

Infinitely many (There is a free variable)



12) Solve the matrix equation below. (20 points)

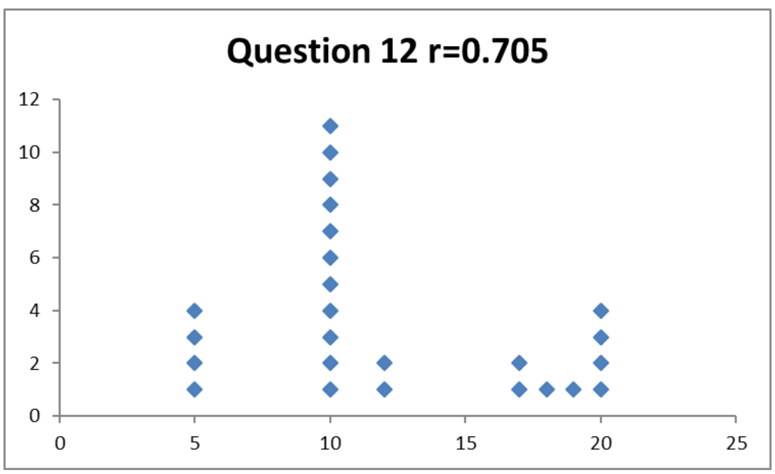
$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & 1 & 1 \end{bmatrix} \underset{R_2 \to R_2 - R_1}{\sim_R} \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & -4 \end{bmatrix} \underset{R_2 \to -R_2}{\sim_R} \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 4 \end{bmatrix} \underset{R_1 \to R_1 - 2R_2}{\sim_R} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_3 \\ -4x_3 \\ x_3 \end{bmatrix}$$

The solution set is:

$$\left\{ \begin{bmatrix} 3s \\ -4s \\ s \end{bmatrix} : s \in \mathbb{R} \right\}$$



13) Row reduce the matrix below to reduced echelon form. (15 points)

$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 3 & 6 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 3 & 6 & 9 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 2 & 1 & 0 \\ 0 & 3 & 6 & 9 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -3 & -4 \\ 0 & 3 & 6 & 9 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 3 & 6 & 9 \\ 0 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $\quad$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$ $\quad$ $R_4 \leftrightarrow R_2$

$$\sim_R \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to \frac{1}{3}R_2$ $\quad$ $R_3 \to R_3 + 2R_2$ $\quad$ $R_2 \to R_2 - 2R_3$, $R_1 \to R_1 + R_3$