

For the problems on this page, use the matrix A below.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 11 & 8 \\ 1 & 2 & 3 & 2 \end{bmatrix} \quad A \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1) Find the row space of the matrix A . (4 points)

2) Find the column space of the matrix A . (4 points)

3) Find the null space of the matrix A . (8 points)

4) Is $\begin{bmatrix} 4 \\ 8 \\ 2 \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 11 \\ 3 \end{bmatrix}$? Why or why not? (6 points)

5) What is the rank of A ? (4 points)

For the problems on this page, use the bases below. Write a formula for your answers, please do not perform the arithmetic.

$$B_1 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 7 \end{bmatrix} \right\}$$

6) Given $\vec{x}_S = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}_S$, what is \vec{x}_{B_1} ? (4 points)

7) Given $\vec{x}_{B_1} = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}_{B_1}$, what is \vec{x}_S ? (4 points)

8) Given $\vec{x}_{B_1} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}_{B_1}$, what is \vec{x}_{B_2} ? (8 points)

9) Find the determinant of the product below. Please perform the arithmetic. (5 points)

$$\begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}$$

10) Find the determinant of the matrix below. Your answer may be a formula as long as it does not involve any determinants. (The basic 4 operations (+ - × ÷) only) (5 points)

$$\begin{bmatrix} 1 & 4 & 6 \\ 7 & 9 & 3 \\ 5 & 8 & 2 \end{bmatrix}$$

11) Find the determinant of the matrix below. Please perform the arithmetic. (10 points)

$$\begin{bmatrix} 1 & 0 & 6 & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Suppose A is a 7×7 matrix such that $A\vec{x} = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4]^T$ has no solutions, but $A\vec{x} = \vec{0}$ has multiple solutions. Answer the following questions.

12) How many solutions does $A\vec{x} = \vec{0}$ have? (2 points)

13) Is A invertible? (2 points)

14) What is the maximum number of pivots A can have? (2 points)

15) What is the maximum number of free variables $A\vec{x} = \vec{0}$ can have? (2 points)

16) What is the maximum rank A can have? (2 points)

17) Are the columns of A linearly independent? (2 points)

Suppose A is a 6×8 matrix such that $A\vec{x} = [1 \ 0 \ 0 \ 0 \ 0 \ 4]^T$ has no solutions, but $A\vec{x} = \vec{0}$ has multiple solutions. Answer the following questions.

18) How many solutions does $A\vec{x} = \vec{0}$ have? (2 points)

19) What is the maximum number of pivots A can have? (2 points)

20) What is the maximum number of free variables $A\vec{x} = \vec{0}$ can have? (2 points)

21) What is the maximum rank A can have? (2 points)

22) Are the rows of A linearly independent? (2 points)

23) Given the matrix A below, find the corresponding system of homogeneous linear equations. (4 points)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 11 & 8 \\ 1 & 2 & 3 & 2 \end{bmatrix} \quad A \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

24) Row reduce the matrix below to reduced echelon form. (12 points)

$$\begin{bmatrix} 2 & 0 & 6 & 4 \\ 1 & 1 & 4 & 2 \\ 0 & 5 & 5 & 6 \end{bmatrix}$$