Name ____ Test 2, Fall 2019

For the problems on this page, use the matrix A below.

$$A=\begin{bmatrix}1&2&3&4\\5&6&11&8\\1&2&3&2\end{bmatrix}\qquad A\sim_R\begin{bmatrix}1&0&1&0\\0&1&1&0\\0&0&0&1\end{bmatrix}$$

1) Find the row space of the matrix A. (4 points)

$$span(\{[1\quad 2\quad 3\quad 4],[5\quad 6\quad 11\quad 8],[1\quad 2\quad 3\quad 2]\})$$

2) Find the column space of the matrix A. (4 points)

$$span\left(\left\{\begin{bmatrix}1\\5\\1\end{bmatrix},\begin{bmatrix}2\\6\\2\end{bmatrix},\begin{bmatrix}4\\8\\2\end{bmatrix}\right\}\right)$$

3) Find the null space of the matrix A. (8 points)

$$span\left(\left\{\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix}\right\}\right)$$

4) Is $\begin{bmatrix}4\\8\\2\end{bmatrix}$ in the span of $\begin{bmatrix}1\\5\\1\end{bmatrix}$, $\begin{bmatrix}2\\6\\2\end{bmatrix}$, and $\begin{bmatrix}3\\11\\3\end{bmatrix}$? Why or why not? (6 points)

It is not. The pivot in the 4th column above tells us that the vector $\begin{bmatrix}4\\8\\2\end{bmatrix}$ is linearly independent from the first three.

5) What is the rank of A? (4 points)

3

For the problems on this page, use the bases below. Write a formula for your answers, please do not perform the arithmetic.

$$B_1 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right\} \qquad B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 7 \end{bmatrix} \right\}$$

6) Given $\vec{x}_S = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}_S$, what is $\vec{x}_{B_1}$? (4 points)

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}_S$$

7) Given $\vec{x}_{B_1} = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}_{B_1}$, what is $\vec{x}_S$? (4 points)

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}_{B_1}$$

8) Given $\vec{x}_{B_1} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}_{B_1}$, what is $\vec{x}_{B_2}$? (8 points)

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 0 \\ 1 & 1 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}_{B_1}$$

9) Find the determinant of the product below. Please perform the arithmetic. (5 points)

$$\begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}$$

$$(3)(1)(-1)^2(6)(1) = 18$$

10) Find the determinant of the matrix below. Your answer may be a formula as long as it does not involve any determinants. (The basic 4 operations $(+ - \times \div)$ only) (5 points)

$$\begin{bmatrix} 1 & 4 & 6 \\ 7 & 9 & 3 \\ 5 & 8 & 2 \end{bmatrix}$$

$$1\begin{vmatrix} 9 & 3 \\ 8 & 2 \end{vmatrix} - 4\begin{vmatrix} 7 & 3 \\ 5 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & 9 \\ 5 & 8 \end{vmatrix} = (9 \cdot 2 - 8 \cdot 3) - 4(14 - 15) + 6(7 \cdot 8 - 5 \cdot 9)$$

11) Find the determinant of the matrix below. Please perform the arithmetic. (10 points)

$$\begin{bmatrix} 1 & 0 & 6 & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$2\begin{vmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 2 \cdot 1 \begin{vmatrix} 1 & 0 & 4 \\ 3 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2 \cdot 1 \cdot 1 \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 2 \cdot 1 \cdot 1 \cdot 3 = 6$$

Suppose A is a 7×7 matrix such that $A\vec{x} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4]^T$ has no solutions, but $A\vec{x} = \vec{0}$ has multiple solutions. Answer the following questions.

First note that this information tells us that in row reduced echelon form (1) there is a row of zeroes and (2) there is free variable. Because the matrix is square, these are equivalent statements.

12) How many solutions does $A\vec{x} = \vec{0}$ have? (2 points)

∞

13) Is A is invertible? (2 points)

No

14) What is the maximum number of pivots A can have? (2 points)

6

15) What is the maximum number of free variables $A\vec{x} = \vec{0}$ can have? (2 points)

7

16) What is the maximum rank A can have? (2 points)

6

17) Are the columns of A linearly independent? (2 points)

No

Suppose A is a 6×8 matrix such that $A\vec{x} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4]^T$ has no solutions, but $A\vec{x} = \vec{0}$ has multiple solutions. Answer the following questions.

First note that this information tells us that in row reduced echelon form (1) there is a row of zeroes and (2) there is free variable. Because the matrix is not square, these are not equivalent statements.

18) How many solutions does $A\vec{x} = \vec{0}$ have? (2 points)

∞

19) What is the maximum number of pivots A can have? (2 points)

5

20) What is the maximum number of free variables $A\vec{x} = \vec{0}$ can have? (2 points)

8

21) What is the maximum rank A can have? (2 points)

5

22) Are the rows of A linearly independent? (2 points)

No

23) Given the matrix A below, find the corresponding system of homogeneous linear equations. (4 points)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 11 & 8 \\ 1 & 2 & 3 & 2 \end{bmatrix} \qquad A \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 5x_1 + 6x_2 + 11x_3 + 8x_4 &= 0 \\ x_1 + 2x_2 + 3x_3 + 2x_4 &= 0 \end{aligned}$$

24) Row reduce the matrix below to reduced echelon form. (12 points)

$$\begin{bmatrix} 2 & 0 & 6 & 4 \\ 1 & 1 & 4 & 2 \\ 0 & 5 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 6 & 4 \\ 1 & 1 & 4 & 2 \\ 0 & 5 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 & 2 \\ 1 & 1 & 4 & 2 \\ 0 & 5 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to \frac{1}{2}R_1$ $\quad$ $R_2 \to R_2 - R_1$ $\quad$ $R_3 \to R_3 - R_1$ $\quad$ $R_3 \to \frac{1}{6}R_3$ $\quad$ $R_1 \to R_1 - 2R_3$